COMBINED FORCED AND FREE TURBULENT CONVECTION IN A VERTICAL TUBE WITH VOLUME HEAT SOURCES AND CONSTANT WALL HEAT ADDITION

by

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ABSTRACT

An analytical investigation was made of combined forced and free turbulent convection in a circular vertical tube with volume heat sources and constant wall heat addition.

The IBM 1620 digital computers located at The George Washington University and also at the United States Naval Academy were used to solve the basic equations in which the parameters are: Prandtl number Pr; Rayleigh number Ra; friction Reynolds number Re*; and the volume heat source parameter F. For fixed values of these parameters the solution gave the fully developed velocity profile, the temperature profile, and the pressure drop. The program also solved for the mixed-mean-to-wall temperature difference, Nusselt number, and Reynolds number. The following values of these parameters were investigated: Pr = 1, 10, 100; Ra = 0, 3^4, 5^4, 10^4; Re* = 10^3, 10^4, 10^5; and F = .1, .5, 1, 10.

The results of laminar flow problems were investigated to check the validity of the program and they compared favorably with known previous results. In the case of pure forced convection, laminar flow with a negligible volume heat source, the velocity profile and the Nusselt number checked exactly with Dr. Ojalvo's results for the same problem with no volume heat sources.

The results for turbulent heat transfer in combined forced and free convection with volume heat sources showed that an increase in Prandtl number had a smaller effect than an increase in Re* on increasing Nusselt number. The volume heat source parameter F had negligible effect on Nu. Rayleigh number had no effect on Nu. The pressure-drop parameter C increased approximately an order of magnitude as Re* increased an order of magnitude. Prandtl number Pr, Ra, and F had no effect on C for Ra < 625. When Ra > 625 and F > 1, increasing Pr decreased C and increasing Ra increased C; and for Ra > 625 and F < 1, increasing Pr increased C and increasing Ra decreased C. Decreasing F for Ra > 625 decreased C. Increasing Pr had less effect than increasing Re* on decreasing the mixed-mean-to-wall temperature difference ϕ_m. Increasing Ra had negligible effect on ϕ_m. Decreasing F increased ϕ_m.

The velocity in the center of the tube lowered and became less positive as Pr, Re*, or volume heat sources were increased. When F < 1, increasing Ra flattened the velocity profile.

Increasing Re* or Pr flattened the temperature difference profile ϕ. Increasing Ra had negligible effect on ϕ. Decreasing F increased ϕ and when F = 1, the temperature difference profile passed through zero.

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CHAPTER I

INTRODUCTION

The first theoretical investigations into combined forced and free laminar flow in a vertical tube were conducted by Ostroumov[10*] and Hallman[3]. In addition to predicting velocity and temperature profiles, Hallman extended the analysis to include calculation of Nusselt numbers and pressure drops. He considered cases in which volume heat sources were either present or not present. He also conducted an experimental investigation[3] of the problem for laminar flow.

The first theoretical investigation into combined forced and free turbulent convection in a vertical tube was conducted by Ojalvo and Grosh[8]. The analysis did not consider cases with volume heat sources present. All cases of laminar heat transfer, pure forced convection or combined forced and free convection, checked exactly with known results. Cases of pure forced convection turbulent flow were not in agreement with known results apparently because too high a value of eddy viscosity was used in the buffer zone and in part of the turbulent core. The

*Superscript numbers refer to similarly numbered references in the Bibliography.

results of combined forced and free turbulent convection were presented.

Sackett[13] presented data for combined forced and free turbulent convection in a vertical tube with a new relationship for the eddy diffusivity of momentum. The results for turbulent flow cases indicated some improvement but did not compare favorably with Dr. Ojalvo's results. The most likely cause was attributed to the reworked computer program.

Jackson[4] conducted an investigation to improve the relationship for the eddy diffusivity of momentum. The results of his study are used in the present study.

The present analysis investigates the transfer of uniform thermal energy from the wall of a round tube and uniform thermal energy from volume heat sources to a fluid flowing vertically upwards in the tube. Both forced and free convection exist in the steady turbulent flow. Only the case of fully developed flow is considered and compared with known results.

In reworking the original computer program, an error was found that would affect Dr. Ojalvo's results for combined forced and free turbulent convection. This error did not occur in Sackett's program, thus the apparent cause for the differences in their results.

The analysis has practical applications in the fields of nuclear reactors and heat exchangers. Its solution will show the effect of uniform volume heat sources on combined forced and free turbulent convection problems.

CHAPTER II

ANALYSIS

The problem to be analyzed is combined forced and free turbulent convection with uniform volume heat sources in a vertical circular tube whose axis is parallel to the direction of the body force. There is to be a net through-flow.

Assumptions

In addition to the description of the problem given above, the following assumptions are made:

1. Axial symmetry exists for the momentum and heat transfer.
2. All fluid properties are constant, except density in the expression for body force. A mean density ρ_m* is used for all other density terms.
3. Viscous dissipation and axial heat conduction are negligible compared with the heat conduction in the radial direction.
4. There is uniform wall heat flux.
5. The velocity and temperature profiles are fully developed. There are no radial or angular velocity components.
6. There is single-phase flow.

*The Nomenclature is given in Appendix A.

7. Flow is steady, turbulent and incompressible.

8. The eddy diffusivity of momentum is given by Jackson's equations. (See Appendix B.)

Basic Equations

The basic equations employed in this analysis are the continuity, Navier-Stokes, and energy equations in cylindrical coordinates[14]. An equation of state is also used.

On the basis of the above assumptions and description of the problem, the continuity equation

$$\frac{\partial \rho}{\partial T} + \nabla \cdot (\rho \bar{V}) = 0$$

reduces to

$$\frac{\partial u}{\partial x} = 0 \quad , \tag{1}$$

the Navier-Stokes (Momentum) equations

$$\rho \frac{d\bar{V}}{dT} = \bar{B}\rho - \nabla p + \mu \nabla^2 \bar{V} + \frac{1}{3}\mu \nabla (\nabla \cdot \bar{V})$$

reduces to

$$\frac{\partial p}{\partial x} + \rho \frac{g}{g_c} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\left[\frac{\mu}{g_c} + \frac{\rho_m \epsilon_M}{g_c}\right]\frac{\partial u}{\partial r}\right) \tag{2}$$

$$\frac{\partial p}{\partial r} = \frac{\partial p}{\partial \psi} = 0 \quad , \tag{3}$$

and the energy equation

$$\rho c_p \frac{dt}{dT} = \nabla \cdot k \nabla t + q'''$$

reduces to

$$\rho_m c_p u \frac{\partial t}{\partial x} - q''' = \frac{1}{r}\frac{\partial}{\partial r}\left(r\left[k + \rho_m c_p \epsilon_H\right]\frac{\partial t}{\partial r}\right) \quad . \tag{4}$$

The equation of state is developed from an expansion of ρ in a Taylor series in t about the reference temperature t_w, and is:[8]

$$\rho = \rho_w\left[1 - \beta(t - t_w)\right] . \tag{5}$$

Development of Equations

Equation (1) indicates that

$$u = u\,(r), \tag{6}$$

and equations (3) indicate that

$$p = p\,(x). \tag{7}$$

If equations (5), (6), and (7) are utilized, equation (2) can be written as

$$\frac{1}{r}\frac{d}{dr}\left(r\left[\frac{\mu}{g_c} + \frac{\rho_m \epsilon_m}{g_c}\right]\frac{du}{dr}\right) + \rho_w \beta \frac{g}{g_c}(t - t_w) = \frac{dp}{dx} + \rho_w \frac{g}{g_c} . \tag{8}$$

The boundary condition of uniform wall heat flux plus the assumptions of constant specific heat, uniform volume heat sources, and a fully developed temperature profile require that

$$\frac{\partial t}{\partial x} = A \qquad \text{(a constant).} \tag{9}$$

Equation (9) is developed in Appendix C. A new variable θ is introduced and defined as

$$\theta = \theta(r) \equiv t(x, r) - t\left(x, \frac{D}{2}\right) = t - t_w . \tag{10}$$

In terms of equation (10), equations (4) and (8) become, respectively,

$$\rho_m c_p u A - q''' = \frac{1}{r}\frac{\partial}{\partial r}\left(r\left[k + \rho_m c_p \epsilon_H\right]\frac{d\theta}{dr}\right) \qquad (11)$$

and

$$\frac{1}{r}\frac{d}{dr}\left(r\left[\frac{\mu}{g_c} + \frac{\rho_m \epsilon_M}{g_c}\right]\frac{du}{dr}(r) + \rho_w \beta \frac{g}{g_c}\theta(r) = \frac{dp}{dx}(x) + \rho_w \frac{g}{g_c} \qquad (12)$$

Although the density ρ_w in equation (12) is actually a function of x, by considering the solutions of equations (11) and (12) to obtain u and θ at a fixed value of x, we can consider ρ_w constant[8]. Our assumption of fully developed flow and heat transfer is consistant with these results. Each side of equation (12) may be set equal to some constant since we have separated the variables[5]. Thus,

$$\frac{dp}{dx} + \rho_w \frac{g}{g_c} = -\frac{32 u_m \mu C}{D^2 g_c} \quad (\text{a constant}) \qquad (13)$$

and

$$\frac{1}{r}\frac{d}{dr}\left(r\left[\frac{\mu}{g_c} + \frac{\rho_m \epsilon_M}{g_c}\right]\frac{du}{dr}\right) + \rho_w \beta \frac{g}{g_c}\theta = -\frac{32 u_m \mu C}{D^2 g_c}. \qquad (14)$$

The pressure-drop parameter C in these equations takes on the value of unity for the special case of pure forced-convection laminar low, as described by Hallman[3].

In order to solve the set of problems described, equations (11) and (14) are nondimensionalized by using the following dimensionless terms:

$\eta \equiv 2r/D$	radius	(15)
$\phi \equiv 16k\theta/q'''D^2$	temperature difference	(16)
$U \equiv u/u_m$	velocity	(17)
$Ra \equiv \rho_m \rho_w \beta c_p A D^4/16\mu k$	Rayleigh number	(18)
$F \equiv \rho_m u_m c_p A/q'''$	volume heat source	(19)

Equation (19) is discussed in Appendix F.

Thus, equation (11) becomes

$$\frac{1}{\eta}\frac{d}{d\eta}\left(\eta\left[1+\frac{\epsilon_H}{\alpha_m}\right]\frac{d\phi}{d\eta}\right) = 4FU - 4 \quad , \tag{20}$$

and equation (14) becomes

$$\frac{1}{\eta}\frac{d}{d\eta}\left(\eta\left[1+\frac{\epsilon_M}{\nu}\right]\frac{dU}{d\eta}\right) = -\frac{Ra\,\phi}{4F} - 8C \quad . \tag{21}$$

In accordance with assumption (8), we will use

$$\frac{\epsilon_H}{\epsilon_M} \equiv \sigma = \frac{6}{\pi^2}\sum_{N=1}^{\infty}\frac{1}{N^2 \exp(.01N^2/P_r)} \tag{22}$$

as given by Lykoudis[6]. Equation (20) may thus be written as

$$\frac{1}{\eta}\frac{d}{d\eta}\left(\eta\left[1+\sigma Pr\frac{\epsilon_M}{\nu}\right]\frac{d\phi}{d\eta}\right) = 4FU - 4 \tag{23}$$

Boundary Conditions

The following boundary conditions are used:

$$U(1) = 0 \tag{24}$$

$$\phi(1) = 0 \tag{25}$$

$$\frac{dU(0)}{d\eta} \equiv U'(0) = 0 \tag{26}$$

$$\frac{d\phi(0)}{d\eta} \equiv \phi'(0) = 0 \tag{27}$$

These boundary conditions come from the physical problem. Equations (24) and (25) state that the fluid velocity and temperature at the wall of the tube ($\eta = 1$) are equal, respectively, to the wall velocity and wall temperature. Equations (26) and (27) come from the fact that heat and momentum are not transferred across the center line ($\eta = 0$) of the tube, due to axial symmetry, resulting in a zero slope for the temperature and velocity profiles at the center line[7].

Discussion of Equations

Equation (21), the momentum equation, and equation (23), the energy equation, are to be solved by a 1620 IBM digital computer for U and ϕ as functions of η . All boundary conditions are to be satisfied. The value of the pressure-drop

parameter C may also be obtained if we use the following form of the continuity equation:

$$U_m = 2\int_0^1 U\eta\, d\eta = 1 \quad . \tag{28}$$

Three dimensionless parameters in our equations are the Prandtl number Pr, the Rayleigh number Ra, and the volume heat source parameter F. The Prandtl number determines the relationship between the velocity and temperature distributions. For the special case of Pr = 1, both profiles are the same. Rayleigh number Ra = Gr Pr where the Grashof number Gr represents a ratio of buoyant forces to viscous forces[12]. Thus, free-convection predominates for large Rayleigh numbers and for the special case of Ra = 0, pure forced-convection is represented. The significance of variations in the magnitude of the volume heat source parameter F is discussed in Appendix F.

Empirical equations for $\frac{\epsilon_M}{\nu}$ are given in Appendix B. They are:

$$\frac{\epsilon_M}{\nu} = \frac{\eta}{1-.005\,Re^*(1-\eta)[4+\sqrt{9-.025\,Re^*(1-\eta)}]} - 1 \quad \text{for } 1-\frac{60}{Re^*} \leq \eta \leq 1 \tag{29}$$

$$\frac{\epsilon_M}{\nu} = .2\,Re^*\eta(1-\eta) - 1 \quad \text{for } \frac{1}{10} \leq \eta < 1-\frac{60}{Re^*} \tag{30}$$

$$\frac{\epsilon_M}{\nu} = 9\,Re^*/500 - 1 \quad \text{for } 0 \leq \eta\,\frac{1}{10}, \tag{31}$$

where $Re^* \equiv Du^*/\nu$, friction Reynolds number (32)

and $u^* \equiv \sqrt{\tau_w g_c/\rho_w}$, friction velocity (33)

Thus, a fourth parameter, the friction Reynolds number Re*, is introduced. In this analysis, when Re* = 0, the ratio $\frac{\epsilon_M}{\nu}$ is set equal to zero, thus assisting in checking the validity of the program by comparison with laminar flow problem results.

Extension of Results

The solution of equations (21), (23), and (28) will yield U and ϕ as functions of η for a given set of values of the parameters: Ra, Pr, Re*, and F. The pressure-drop parameter C will also be obtained.

These results may be extended by calculating the following useful quantities: the dimensionless mixed-mean-to-wall temperature difference ϕ_m; the Nusselt number Nu; and the Reynolds number Re.

Hallman[3] gives equations for the first two of these quantities. They are:

$$\phi_m \equiv 2 \int_0^1 \phi U \eta \, d\eta \qquad (34)$$

and

$$Nu \equiv \frac{4}{\phi_m} (1 - F) \; ; \qquad (35)$$

see Appendix D.

An expression for the Reynolds number is derived in

Appendix E. The result is

$$Re \equiv \frac{\pm (Re^*)^2}{8C + Ra\,\phi_m/4F} \quad , \tag{36}$$

where the + sign is for upward flow and the - sign is for net downward flow.

CHAPTER III

METHOD OF SOLUTION

The method of solution is based on the use of a digital computer. This method will decrease considerably the amount of hand calculations required in the solution of our problems. The possibility of errors are thus reduced. This method of solution will also allow us to investigate many different problems and compare some of these results with known analytical and experimental data.

The independent parameters used are Ra, Re*, Pr, and F. The Rayleigh number is chosen to measure the extent of the free-convective effect on the flow. The friction Reynolds number determines the value of $\frac{\epsilon_M}{\nu}$ as a function of η. The Prandtl number determines the value of σ. The volume heat source parameter is chosen to describe the thermal energy convected downstream to the heat generated in the fluid. With these input quantities fixed, the method of solution proceeds as follows:

1. $$U_1 = 2\,(1 - \eta^2) \tag{37}$$

is assumed as an initial guess for the velocity profile. This equation satisfies the continuity equation $\int_0^1 U\eta\,d\eta = \frac{1}{2}$,

and the boundary conditions U(1) = 0, and U'(0) = 0. It is the limiting case of the parabolic profile in pure forced-convection laminar flow.

2. Equation (23) is integrated with the aid of boundary condition $\phi'(0) = 0$ to obtain

$$\phi' = \frac{4F\int_0^\eta U_i \eta \, d\eta}{\eta\left[1+\sigma Pr \frac{\epsilon_M}{\nu}\right]} - \frac{2\eta}{\left[1+\sigma Pr \frac{\epsilon_M}{\nu}\right]} \quad (38)$$

3. Equation (38) is integrated using boundary condition $\phi(1) = 0$ to obtain

$$\phi = \int_1^\eta \phi' \, d\eta \quad (39)$$

as a function of η.

4. Equation (21) is integrated using boundary condition U'(0) = 0 to obtain

$$U_2' = -\frac{\frac{Ra}{4F\eta}\int_0^\eta \phi \eta \, d\eta}{\left[1+\frac{\epsilon_M}{\nu}\right]} - \frac{4\eta C}{\left[1+\frac{\epsilon_M}{\nu}\right]} \quad (40)$$

The pressure-drop parameter C must have an output numerical value for the computer. For the limiting case of pure forced-convection laminar flow C = 1.

5. Equation (40) is integrated subject to boundary condition U(1) = 0 to obtain

$$U_2 = \int_1^\eta U_2' \, d\eta \quad (41)$$

as a function of η.

6. The continuity equation (28) is checked to see if equation (41) satisfies it. If U_2 does not satisfy this equation,

the value of C is changed until it is satisfied.

7. The function U_2 is checked against U_1 by comparing $U'_2(1)$ with $U'_1(1)$ to see if they are within 0.1% of each other.

8. Equations

$$\phi_m = 2\int_0^1 \phi U \eta \, d\eta \quad ,$$

$$Nu = \frac{4}{\phi_m}(1-F) \quad ,$$

and $$Re = \frac{\pm (Re^*)^2}{8C + Ra\,\phi_m/4F}$$

are used to calculate ϕ_m, Nu and Re, respectively, if the check in Step 7 is satisfactory. The results are then printed out as given in Step 10 below.

9. If the check in Step 7 is not satisfactory, U_2 is used as an initial assumption, replacing U_1 in going through the procedure again, starting at Step 2.

10. Print out

Title

Problem Number			F
Ra	Re	Pr	σ
η	U_2	U_1	ϕ
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
1	0	0	0
C	ϕ_m	Nu	Re

U_2 and U_1 are both printed out to ensure that the criterion for checking them in Step 7 is valid.

These steps may be shown in a simplified flow diagram for the calculation, Figure 1.

E_2 and Q_2 are both printed out to [illegible]

for checking them in Step 7 is valid.

These steps may be shown in a simplified flow [illegible]

the calculation, Figure 1.

CHAPTER IV

COMPARISON OF RESULTS

Table 1 lists the following calculated results: C, ϕ_m, Re, and Nu for all problems solved by the 1620 digital computer.

TABLE 1

RESULTS FOR ALL COMPLETED PROBLEMS

Problem No.	Input Parameters: Ra	Re*	Pr	F	Calculated Results: C	ϕ_m	Re	Nu
			A. Laminar flow, pure forced convection					
403B	0	0	---	10^3	1.0	-915.85	--	4.36
			B. Laminar flow, combined forced and free					
402B	81	0	---	10	2.4	-9.62	--	3.74
			C. Turbulent flow, pure forced convection					
379	0	10^3	1	.5	7.3	.036	17000	55.4
380	0	10^3	1	1.0	7.3	-.0034	17000	0.0
383	0	10^3	1	10.0	7.3	-.71	17000	50.3
385	0	10^3	10	.5	7.3	.01	17000	196.0
386	0	10^3	10	1.0	7.3	-.00053	17000	0.0
389	0	10^3	10	10.0	7.3	-.194	17000	185.0
391	0	10^3	100	.5	7.3	.002	17000	983.0
392	0	10^3	100	1.0	7.3	-.000064	17000	0.0
395	0	10^3	100	10.0	7.3	-.038	17000	952.0
397	0	10^4	1	.5	53.9	.005	232000	400.0
398	0	10^4	1	1.0	53.9	-.00017	232000	0.0
400B	0	300	1	10.0	2.8	-2.07	4030	17.4
401	0	10^4	1	10.0	53.9	-.093	232000	385.0
403	0	10^4	10	.5	53.9	.0011	232000	1750.0
404	0	10^4	10	1.0	53.9	-.000019	232000	0.0
407	0	10^4	10	10.0	53.9	-.02	232000	1720.0
409	0	10^4	100	.5	53.9	.00025	232000	7900.0

Table 1 (Continued)

Problem No.	Ra	Re*	Pr	F	C	ϕ_m	Re	Nu
		Input Parameters					Calculated Results	
		C. Turbulent flow, pure forced convection (Continued)						
410	0	10^4	100	1.0	53.9	-.000002	232000	0.0
413	0	10^4	100	10.0	53.9	-.0046	232000	7850.0
415	0	10^5	1	.5	430.0	.00063	2900000	3150.0
416	0	10^5	1	1.0	430.0	-.000021	2900000	0.0
419	0	10^5	1	10.0	430.0	-.012	2900000	3040.0
421	0	10^5	10	.5	430.0	.00017	2900000	11700.0
422	0	10^5	10	1.0	430.0	-.0000034	2900000	0.0
425	0	10^5	10	10.0	430.0	-.0031	2900000	11500.0
427	0	10^5	100	.5	430.0	.0001	2900000	18600.0
428	0	10^5	100	1.0	430.0	-.0000014	2900000	0.0
429B	0	10^4	1	10^3	53.9	-10.34	232000	386.0
		D. Turbulent flow, combined forced and free						
55	81	10^3	1	.1	5.8	.067	16600	53.1
56	81	10^3	1	.5	7.1	.036	17000	55.4
57	81	10^3	1	1.0	7.3	-.0036	17000	0.0
60	81	10^3	1	10.0	7.5	-.718	17000	50.0
61	81	10^3	10	.1	6.9	.018	16900	191.0
62	81	10^3	10	.5	7.3	.01	17000	196.0
63	81	10^3	10	1.0	7.3	-.00054	17000	0.0
66	81	10^3	10	10.0	7.4	-.194	17000	185.0
67	81	10^3	100	.1	7.2	.0037	17000	970.0
68	81	10^3	100	.5	7.3	.002	17000	983.0
69	81	10^3	100	1.0	7.3	-.000064	17000	0.0
72	81	10^3	100	10.0	7.3	-.037	17000	952.0
73	81	10^4	1	.1	53.7	.0091	232000	394.0
74	81	10^4	1	.5	53.9	.005	232000	400.0
75	81	10^4	1	1.0	53.9	-.00017	232000	0.0
78	81	10^4	1	10.0	53.9	-.093	232000	385.0
79	81	10^4	10	.1	53.8	.002	232000	1740.0
80	81	10^4	10	.5	53.9	.001	232000	1750.0
81	81	10^4	10	1.0	53.9	-.00002	232000	0.0
84	81	10^4	10	10.0	53.9	-.021	232000	1720.0
85	81	10^4	100	.1	53.9	.00045	232000	7890.0
86	81	10^4	100	.5	53.9	.00025	232000	7900.0
87	81	10^4	100	1.0	53.9	-.000002	232000	0.0
90	81	10^4	100	10.0	53.9	-.0045	232000	7850.0

Table 1 (Continued)

Problem No.	Ra	Re*	Pr	F	C	ϕ_m	Re	Nu
	Input Parameters					Calculated Results		
	D. Turbulent flow, combined forced and free (Continued)							
91	81	10^5	1	.1	427.0	.0011	2920000	3120.0
92	81	10^5	1	.5	430.0	.00063	2900000	3150.0
93	81	10^5	1	1.0	430.0	-.000021	2900000	0.0
96	81	10^5	1	10.0	430.0	-.012	2900000	3040.0
97	81	10^5	10	.1	427.0	.0003	2920000	11700.0
98	81	10^5	10	.5	430.0	.00017	2900000	11700.0
99	81	10^5	10	1.0	430.0	-.0000034	2900000	0.0
102	81	10^5	10	10.0	430.0	-.0031	2900000	11500.0
103	81	10^5	100	.1	427.0	.00019	2920000	18600.0
104	81	10^5	100	.5	430.0	.0001	2900000	18600.0
105	81	10^5	100	1.0	430.0	-.0000014	2900000	0.0
108	81	10^5	100	10.0	430.0	-.0019	2900000	18400.0
164	625	10^3	1	.5	6.0	.036	16700	55.4
165	625	10^3	1	1.0	7.4	-.0035	17000	0.0
169	625	10^3	10	.1	4.0	.019	16200	191.0
170	625	10^3	10	.5	7.0	.010	17300	196.0
171	625	10^3	10	1.0	7.3	-.00056	17000	0.0
174	625	10^3	10	10.0	7.6	-.194	32000	185.0
175	625	10^3	100	.1	6.7	.0037	16900	970.0
176	625	10^3	100	.5	7.2	.002	17100	983.0
177	625	10^3	100	1.0	7.3	-.000064	17000	0.0
180	625	10^3	100	10.0	7.4	-.038	18800	952.0
181	625	10^4	1	.1	52.2	.0091	231000	394.0
182	625	10^4	1	.5	53.7	.005	232000	400.0
183	625	10^4	1	1.0	53.9	-.00017	232000	0.0
187	625	10^4	10	.1	53.5	.002	232000	1740.0
188	625	10^4	10	.5	53.8	.0011	232000	1750.0
189	625	10^4	10	1.0	53.9	-.000019	232000	0.0
192	625	10^4	10	10.0	53.9	-.021	233000	1720.0
193	625	10^4	100	.1	53.8	.00046	232000	7890.0
194	625	10^4	100	.5	53.9	.00025	232000	7910.0
195	625	10^4	100	1.0	53.9	-.000002	232000	0.0
198	625	10^4	100	10.0	53.9	-.0046	232000	7850.0
199	625	10^5	1	.1	427.0	.0011	2920000	3120.0
200	625	10^5	1	.5	430.0	.00063	2900000	3150.0
201	625	10^5	1	1.0	430.0	-.000021	2900000	0.0
205	625	10^5	10	.1	427.0	.0003	2920000	11700.0

Table 1 (Continued)

Problem No.	Ra	Re*	Pr	F	C	ϕ_m	Re	Nu
	Input Parameters					Calculated Results		
	D. Turbulent flow, combined forced and free (Continued)							
206	625	10^5	10	.5	430.0	.00017	2910000	11700.0
207	625	10^5	10	1.0	430.0	-.0000034	2910000	0.0
210	625	10^5	10	10.0	430.0	-.0031	2910000	11500.0
211	625	10^5	100	.1	427.0	.00019	2920000	18600.0
212	625	10^5	100	.5	430.0	.0001	2910000	18600.0
213	625	10^5	100	1.0	430.0	-.0000014	2910000	0.0
216	625	10^5	100	10.0	430.0	-.0019	2910000	18300.0
327	10^4	10^3	1	1.0	8.3	-.0035	17400	0.0
330	10^4	10^3	1	10.0	27.6	-.73	27600	48.7
332	10^4	10^3	10	.5	1.6	.01	15700	196.0
333	10^4	10^3	10	1.0	7.5	-.00054	17100	0.0
336	10^4	10^3	10	10.0	12.7	-.194	18700	185.0
338	10^4	10^3	100	.5	6.2	.002	16700	984.0
339	10^4	10^3	100	1.0	7.4	-.000072	17000	0.0
342	10^4	10^3	100	10.0	8.4	-.038	17300	948.0
343	10^4	10^4	1	.1	27.4	.0091	223000	393.0
344	10^4	10^4	1	.5	51.1	.005	230000	400.0
345	10^4	10^4	1	1.0	54.0	-.0002	231000	0.0
348	10^4	10^4	1	10.0	56.7	-.094	232000	383.0
349	10^4	10^4	10	.1	47.8	.002	230000	1740.0
350	10^4	10^4	10	.5	53.2	.0011	232000	1750.0
351	10^4	10^4	10	1.0	53.9	-.000019	232000	0.0
354	10^4	10^4	10	10.0	54.5	-.021	232000	1720.0
355	10^4	10^4	100	.1	52.5	.00045	231000	7890.0
356	10^4	10^4	100	.5	53.7	.00025	232000	7910.0
357	10^4	10^4	100	1.0	53.9	-.000002	232000	0.0
360	10^4	10^4	100	10.5	54.0	-.0046	232000	7850.0
361	10^4	10^5	1	.1	424.0	.0011	2920000	3120.0
362	10^4	10^5	1	.5	430.0	.00063	2900000	3150.0
363	10^4	10^5	1	1.0	430.0	-.00006	2900000	0.0
366	10^4	10^5	1	10.0	430.0	-.012	2910000	3050.0
367	10^4	10^5	10	.1	426.0	.00003	2920000	11700.0
368	10^4	10^5	10	.5	430.0	.00017	2910000	11700.0
369	10^4	10^5	10	1.0	430.0	-.0000034	2910000	0.0
372	10^4	10^5	10	10.0	430.0	-.0031	2910000	11500.0
373	10^4	10^5	100	.1	427.0	.00019	2920000	18600.0
374	10^4	10^5	100	.5	430.0	.0001	2910000	18600.0
375	10^4	10^5	100	1.0	430.0	-.0000014	2910000	0.0
378	10^4	10^5	100	10.0	430.0	-.0019	2910000	18400.0

Part A consists of the problem in laminar flow, pure forced convection with negligible volume heat sources. Part B consists of a problem in laminar flow, combined forced and free convection with volume heat sources. Part C consists of 28 problems in turbulent flow, pure forced convection with volume heat sources. Part D consists of 100 problems in turbulent flow, combined forced and free convection with volume heat sources.

The IBM solutions to all problems are an attachment to the present analysis. Copies of the Fortran program, the data cards, and the solutions to all problems on IBM cards are in the files of Dr. Ojalvo.

Discussion

The velocity profile and the Nusselt number of the laminar flow problem in part A, Table 1, checked exactly with the results of Ojalvo[8] and Hallman[3]. A negligible volume heat source, F=1000, was used in this problem to check the validity of the program. The laminar flow problem, combined forced and free convection with volume heat sources of part B, Table 1, compared favorably with Hallman. It also compared favorably with Ojalvo even though no volume heat sources were present in his analysis.

Problem 397 was chosen as typical of the problems in part C. This problem was compared with test problem 429B and the universal velocity and temperature profiles of Eckert and Drake[2].

The values of U and ϕ_m in problem 429B with negligible volume heat sources, F=1000, were checked by using the definitions[2]

$$u^+ \equiv u \Big/ \sqrt{\tau_w / \rho_w} \quad , \tag{42}$$

$$y^+ \equiv y \sqrt{\tau_w / \rho_w} \Big/ \nu \quad , \tag{43}$$

and

$$(\Delta t)^+ \equiv \frac{t - t_w}{q''_w \Big/ \rho_m c_p \sqrt{\tau_w / \rho_w}} \tag{44}$$

with the definitions of the various dimensionless quantities to derive the comparison equations

$$u^+ = \frac{Re}{Re^*} (U) \quad , \tag{45}$$

$$y^+ = \frac{1}{2} Re^* (1 - \eta) \quad , \tag{46}$$

and

$$(\Delta t^+) = \frac{1}{4} \frac{Pr\, Re^*}{(1 - F)} (\phi) \quad . \tag{47}$$

Figure 2 shows the comparison of u^+ and $-(\Delta t^+)$ for problem 429B with the universal velocity and temperature profiles. The values of velocity coincide almost exactly up to $y^+ = 100$, then they start to diverge slowly below the universal profile. The values of temperature fall slightly above the universal

temperature profile. The values of problem 397 concide exactly with those of problem 429B except the temperature starts to diverge above the values of problem 429B at $y^{+}=300$. Volume heat sources had negligible effect on the velocity and temperature profiles.

The differences in the values of these two problems and problem 3 of Dr. Ojalvo[8] are attributed to the improved expression for the relative viscosity. A decrease in F from 1000 to .5, that is an increase in the volume heat sources, had very little effect on the profiles.

All of the problems in part C with a Prandtl number of 1 or 10 and with a volume heat source parameter F=10 are plotted in Figure 3, which shows the variation of Nusselt number with Reynolds number. The Dittus-Boelter equation[7] for pure forced-convection turbulent heat transfer is also plotted. In both Prandtl number comparisons, the results of the present analysis compare almost exactly with the curves of the Dittus-Boelter equation. Volume heat sources had negligible effect on the variation of Nusselt number with Reynolds number.

Figures 4 through 10 show results for the problems in turbulent flow, combined forced and free convection. The Nusselt number is plotted against Rayleigh number in Figure 4. The

Nusselt number is increased less than an order of magnitude as Re^* is increased an order of magnitude. Increasing the Prandtl number has less effect on increasing the Nussult number than increasing Re^*. Increasing the Rayleigh number had no effect on Nusselt number. Decreasing F had a very little increasing effect on Nu.

Several test problems were run to determine the limitations of the computer for $Re^*=10^3$. No solutions were obtained on the IBM 1620 computer for $Re^*=10^3$, $F=.1$ or less, and $Ra=625$ or more. Only by decreasing Ra or increasing either Re^* or F would the computer give a solution. A large Ra in combination with a small F (a large volume heat source) would physically appear to be opposing each other. Convective forces predominate in a large Ra, whereas with a small F, considerably more heat is being generated than is being convected downstream.

Figure 5 shows the pressure-drop parameter C plotted against Rayleigh number. C increased approximately an order of magnitude as Re^* increased an order of magnitude. For Rayleigh number less than 625; Ra, F, and Pr had no effect on C. For values of Ra over 625 and $F \geq 1$, increasing Pr decreases C and increasing Ra increases C. If $F<1$, increasing Pr increases C and increasing Ra decreases C. These curves show that C changes

slope for a value of F approximately equal to 1. For a given Re* and Pr the point at which C changes slope is a function of Ra. For small values of F and Re*, and large values of Ra, C would appear to go negative verifying Hallman's results for laminar flow[3]. As Re* is increased the pumping action of large heat sources is reduced considerably.

Figures 6, 7, and 8 show the variation of mixed-mean-to-wall temperature difference with Rayleigh number. Increasing Re* an order of magnitude approximately decreases ϕ_m an order of magnitude. Increasing Ra has no effect on ϕ_m. Increasing Pr has less effect on decreasing ϕ_m than increasing Re*. Finally increasing F has the most effect on decreasing ϕ_m. As shown in Figure 8, ϕ_m passes through zero for F approximately equal to 1. If ϕ_m is plotted against 1/F, as is shown in Figure 9, it is seen that a variation in Ra, Pr, or Re* has little effect on this crossover point. Hallman's curve for laminar flow, Ra=0 is plotted for comparison.

When ϕ_m=0, Nu=∞, according to equation D-3. Figure 10 shows Nusselt number plotted against 1/F for Ra=0, Re*=10^3, and Pr of 1 and 10. An infinite discontinuity occurs whenever ϕ_m=0. From equation D-3 it is seen that Nu=0 for F=1. Hallman's plot for laminar flow, Ra=0 is shown for comparison.

slope for a value of θ [illegible] equals

Ra^* and Pr the point at which [illegible]

Ra. For small values of θ and Ra^*, and large [illegible]

would appear to go negative [illegible]

laminar flow. As Ra^* is increased [illegible]

heat sources is reduced considerably.

Figures 6, 7, and 8 show the [illegible]

wall temperature difference with [illegible]

Ra^* an order of magnitude [illegible]

of magnitude. Increasing θ [illegible]

Pr has less effect on decreasing [illegible]

Finally increasing θ [illegible]

As shown in Figure 7, θ_m [illegible]

mately equal to [illegible]

in Figure 9. [illegible]

little effect on [illegible]

laminar flow. [illegible]

When θ_m [illegible]

shows [illegible]

Pr of 1 and [illegible]

θ_m=0 [illegible]

man's [illegible]

Representative velocity and temperature profiles are shown in Figures 11 through 21.

The effect of varying Re* is shown in Figures 11 through 16 for Pr=10, Ra=625, and for various values of F. As Re* increases from 10^3 to 10^4, the velocity in the center of the tube becomes less positive and the velocity near the wall becomes more positive. As Re* is increased, the temperature profiles flatten out and the slope becomes higher at the wall. Decreasing F, as shown in Figure 21, flattens the velocity profile, whereas the temperature profile decreased considerably and passes through zero at F=1. The effect of increasing F on the velocity and temperature profiles was not as is predicted in appendix F. The action is due to the increased values of Re* investigated in the turbulent flow problems.

Figures 17 and 18 show the effect of various Prandtl numbers for Re*=10^3, Ra=10^4, and F=10. As Prandtl number increases, the velocity and temperature profiles decrease more in the center of the tube than near the wall, thus becoming more flat.

The effect of a variation of Ra for Re*=10^3, Pr=10, and F=.5 is shown in Figures 19 and 20. As Ra is increased from 0 to 10^4 there is very little decrease in the velocity and temperature profiles at the center of the tube.

CHAPTER V

SUMMARY AND CONCLUSIONS

The effect of the parameters Re*, Pr, Ra, and F on Nu, C, ϕ_m, U, and ϕ profiles is summarized in Table 2.

TABLE 2

SUMMARY OF EFFECTS IN COMBINED FORCED AND FREE CONVECTION

Change on for	Nu	C Ra<625	C Ra>625	ϕ_m	U Velocity profile	ϕ Temperature difference profile
F > 1 increasing						
Re*	increase	increase	increase	decrease	flattens curve	flattens curve and makes ϕ approach zero
Pr	increase	no effect	decrease	decrease	lowers velocity in center of tube, making it less positive	same as Re*
Ra	no effect	no effect	increase	slight decrease at high Ra	increase in center of tube	decreases in center of tube

Table 2 (Continued)

Change on for	Nu	C Ra<625	C Ra>625	ϕ_m	U Profile	ϕ Profile
F=1 increasing						
Re*	0	increase	increase	decrease	flattens curve	negligible effect
Pr	0	no effect	decrease	decrease	no effect	negligible effect
Ra	0	no effect	no effect	no effect	no effect	no effect
F<1 increasing						
Re*	increase	increase	increase	decrease	flattens curve	same as F>1
Pr	increase	no effect	increase	decrease	flattens curve	same as F>1
Ra	no effect	no effect	decrease	no effect	flattens curve slightly	negligible effect
decreasing F	slight increase	no effect	decrease	increase	flattens curve	increase, passing through zero, F=1

Conclusions

1. All test problems checked sufficiently close to previous known results to insure the validity of the program.

2. The new expression used for the relative viscosity enabled the universal velocity and temperature profiles to be approached almost exactly in pure forced-convection turbulent heat transfer problems with negligible volume heat sources.

3. Volume heat sources had negligible effect on the velocity profile, but had considerable effect on the temperature profile as would be expected from the development of the equations.

4. The program was unable to solve problems involving high Rayleigh numbers and volume heat sources for low values of Re*.

5. Increasing the various input parameters tend to flatten the velocity and temperature profiles as would be expected.

6. The effect of volume heat sources was reduced considerably by the influence of turbulent flow.

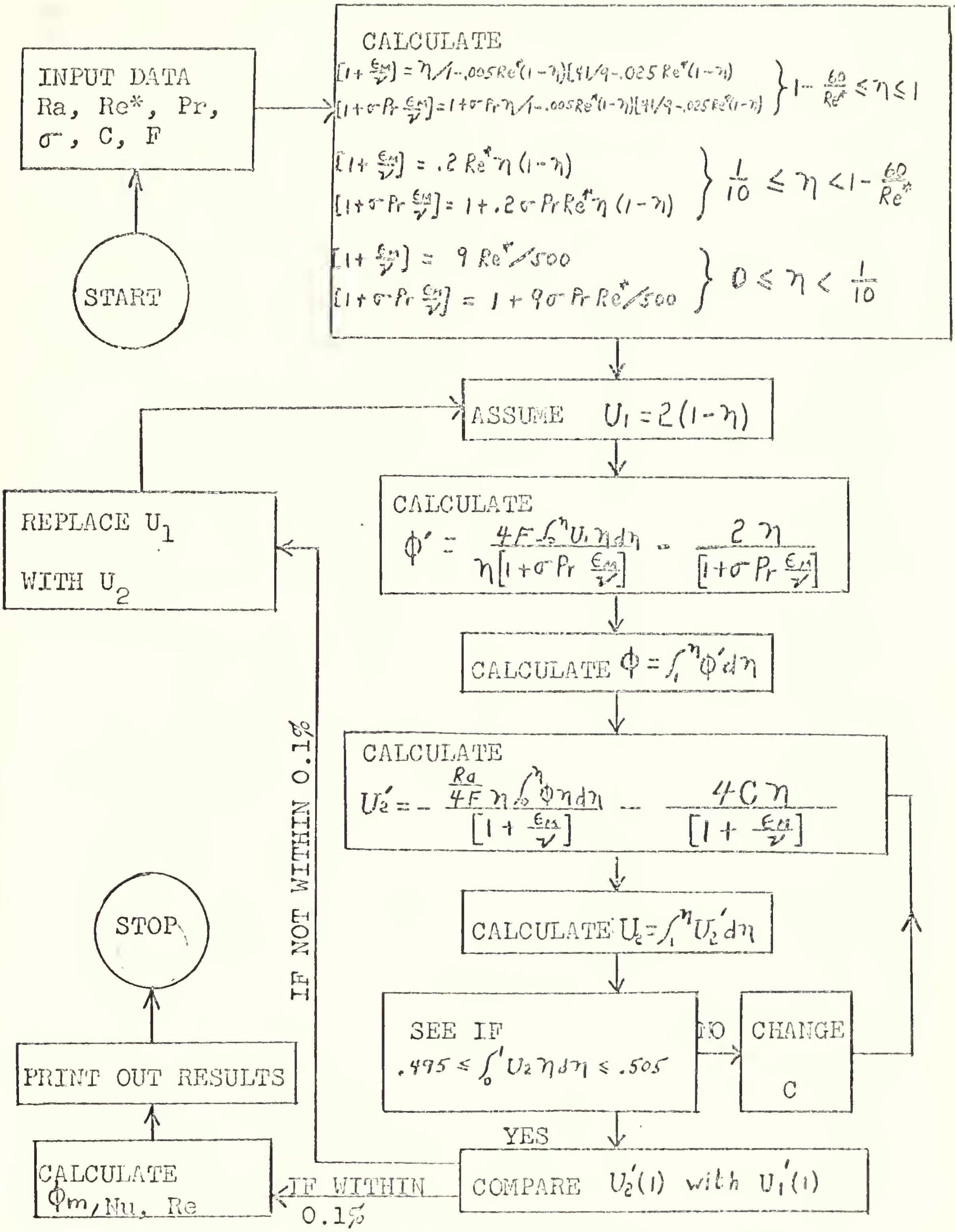

Figure 1. Simplified Flow Diagram of the Calculations

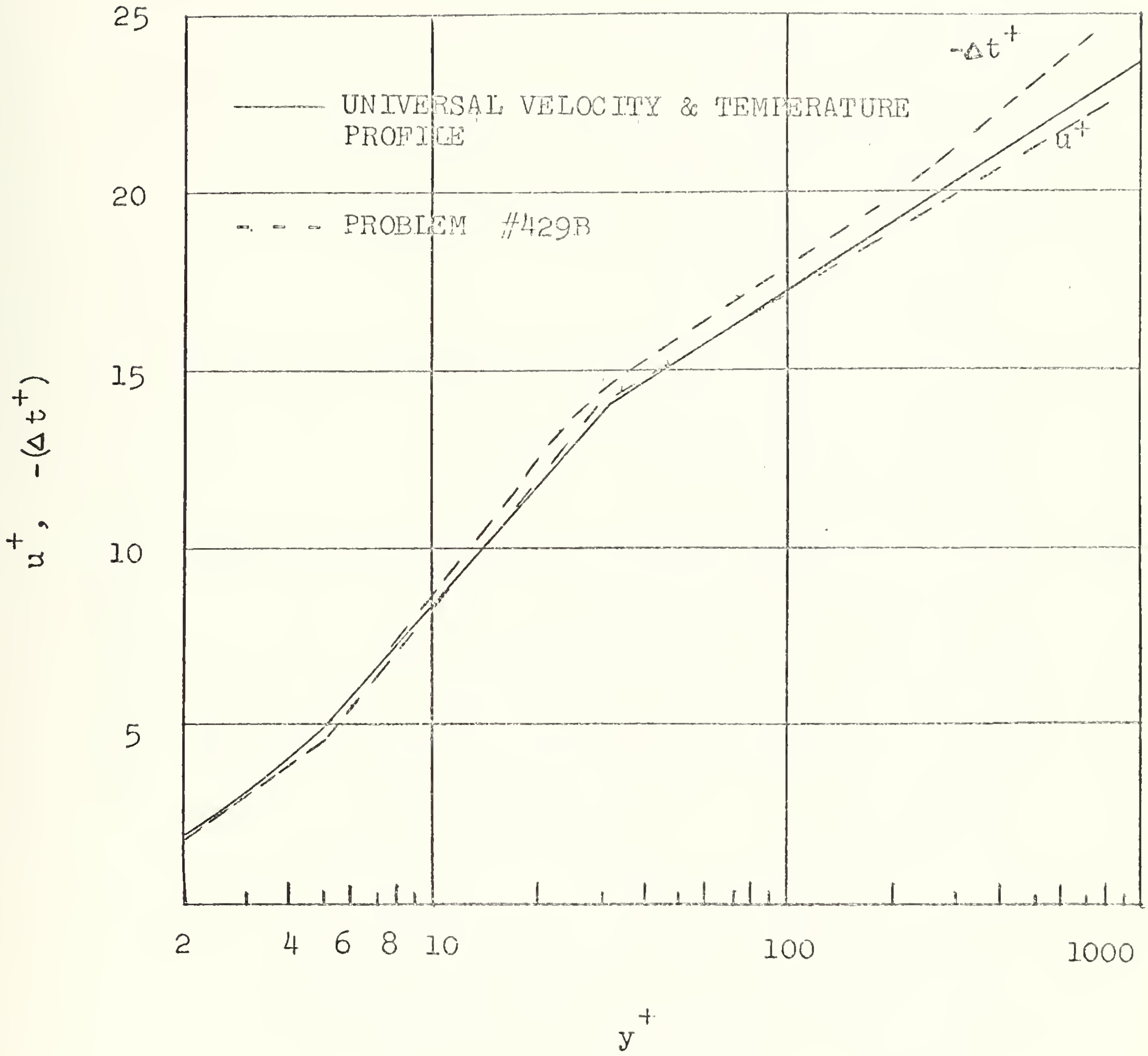

Figure 2. Comparsion of Problem 429B and 397 Results with Universal Velocity and Temperature Profile

Dittus-Boelter Equation

$$Nu = 0.023(Re)^{0.8}(Pr)^{.4}$$

Results of Present analysis for F = 10

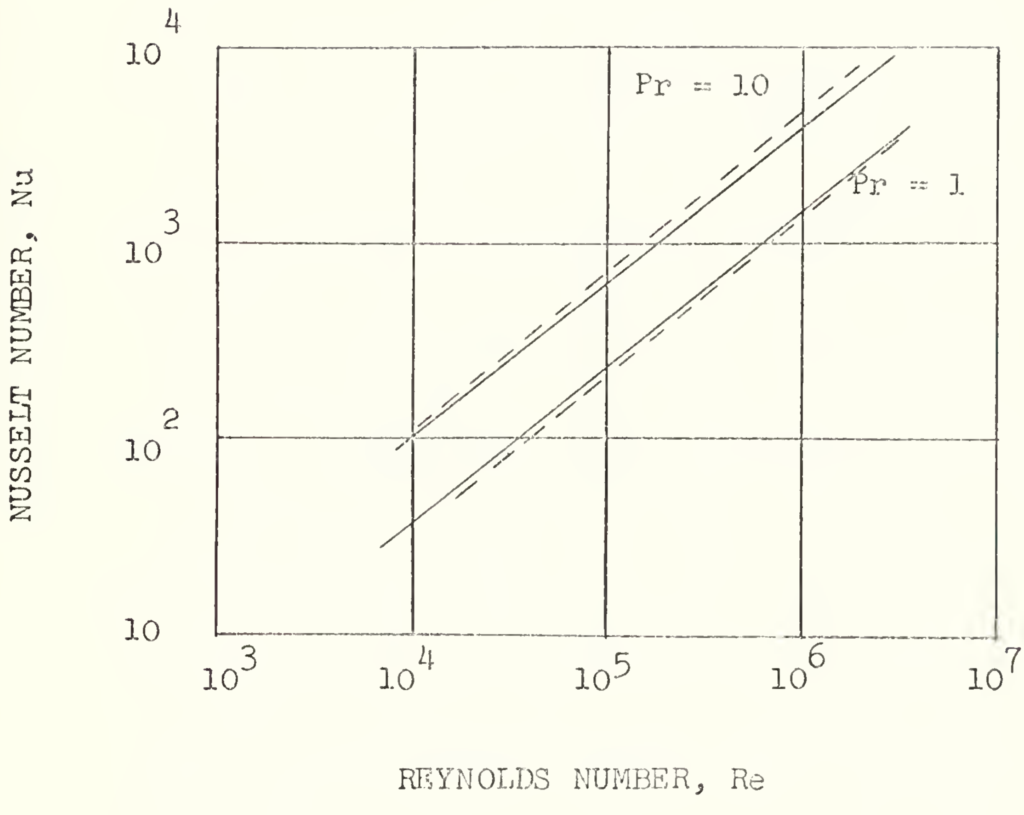

Figure 3. Comparsion of Turbulent Flow, Pure Forced Convection Results with Dittus-Boelter Equation

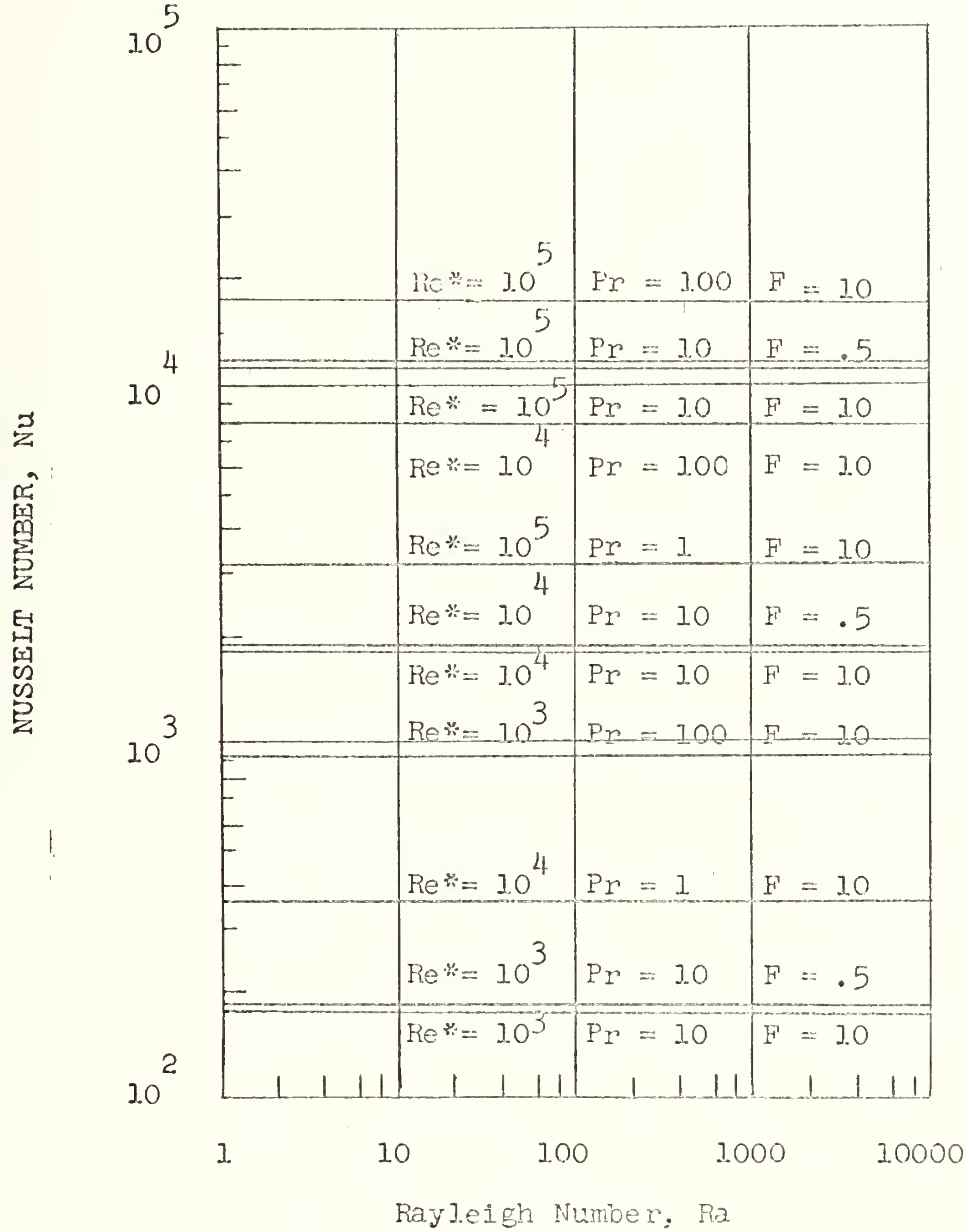

Figure 4. Variation of Nusselt Number with Rayleigh Number

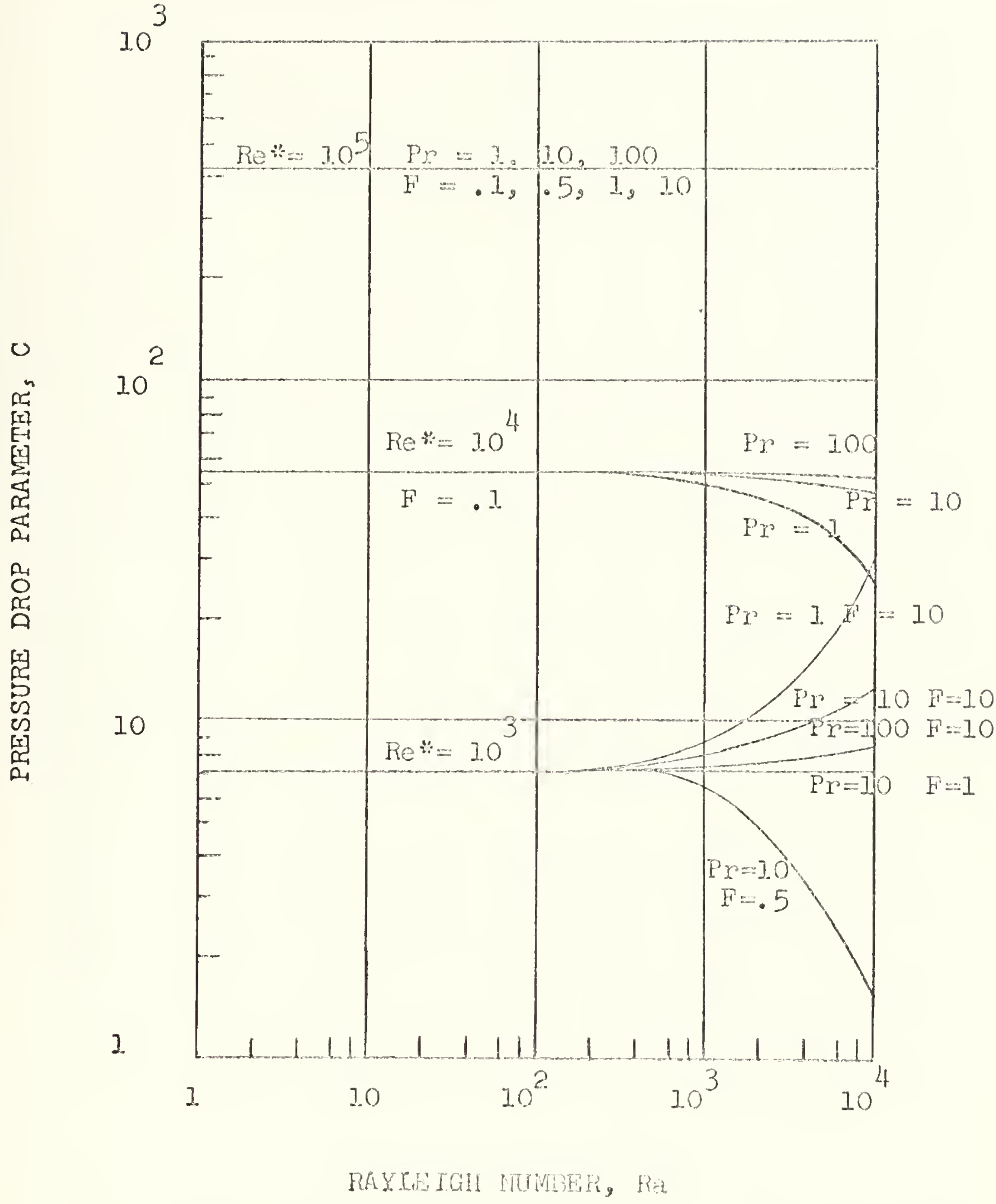

Figure 5. Variation of Pressure Drop Parameter with Rayleigh Number

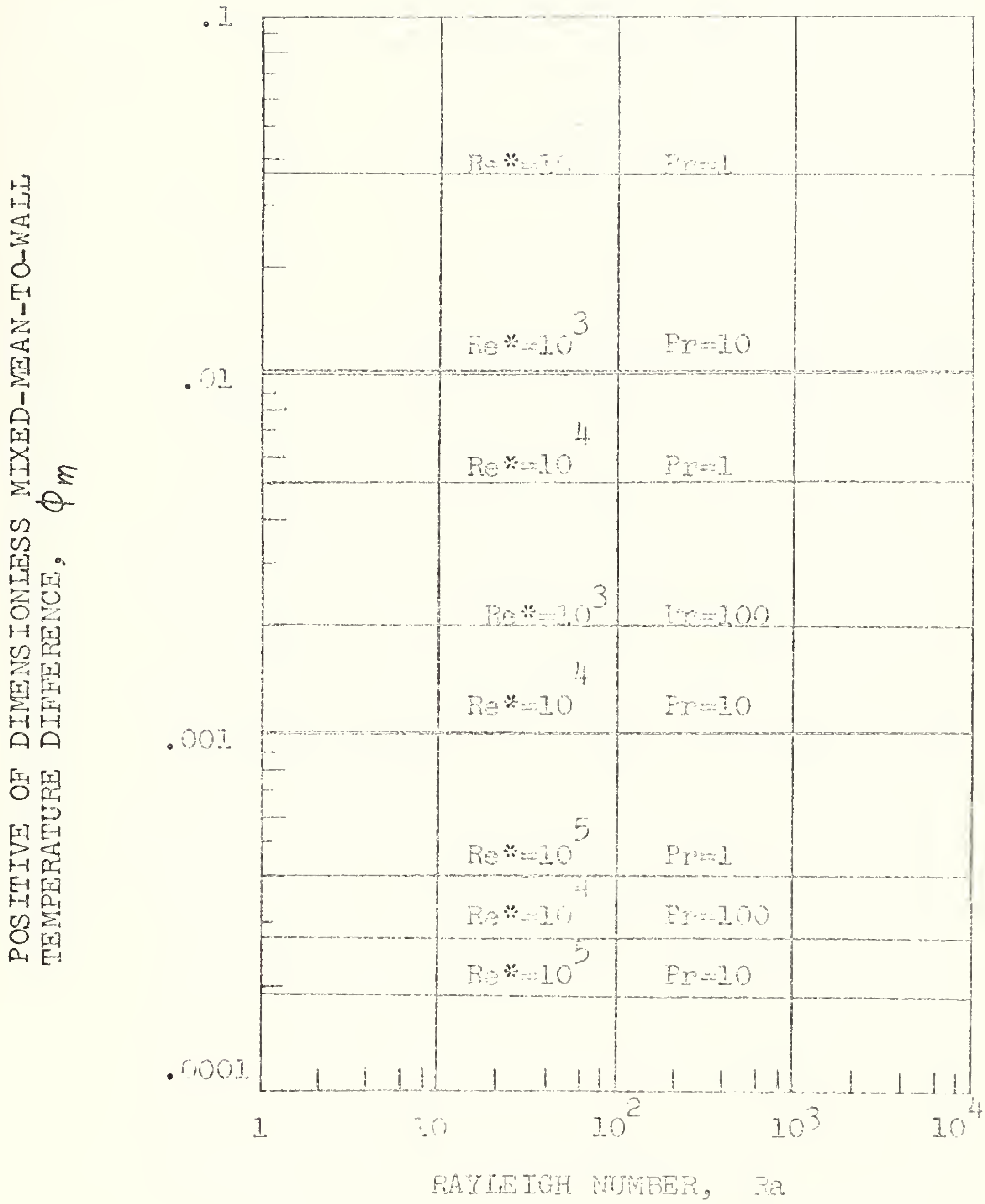

Figure 6. Variation of Positive Mean Temperature Difference with Rayleigh Number for F = .5

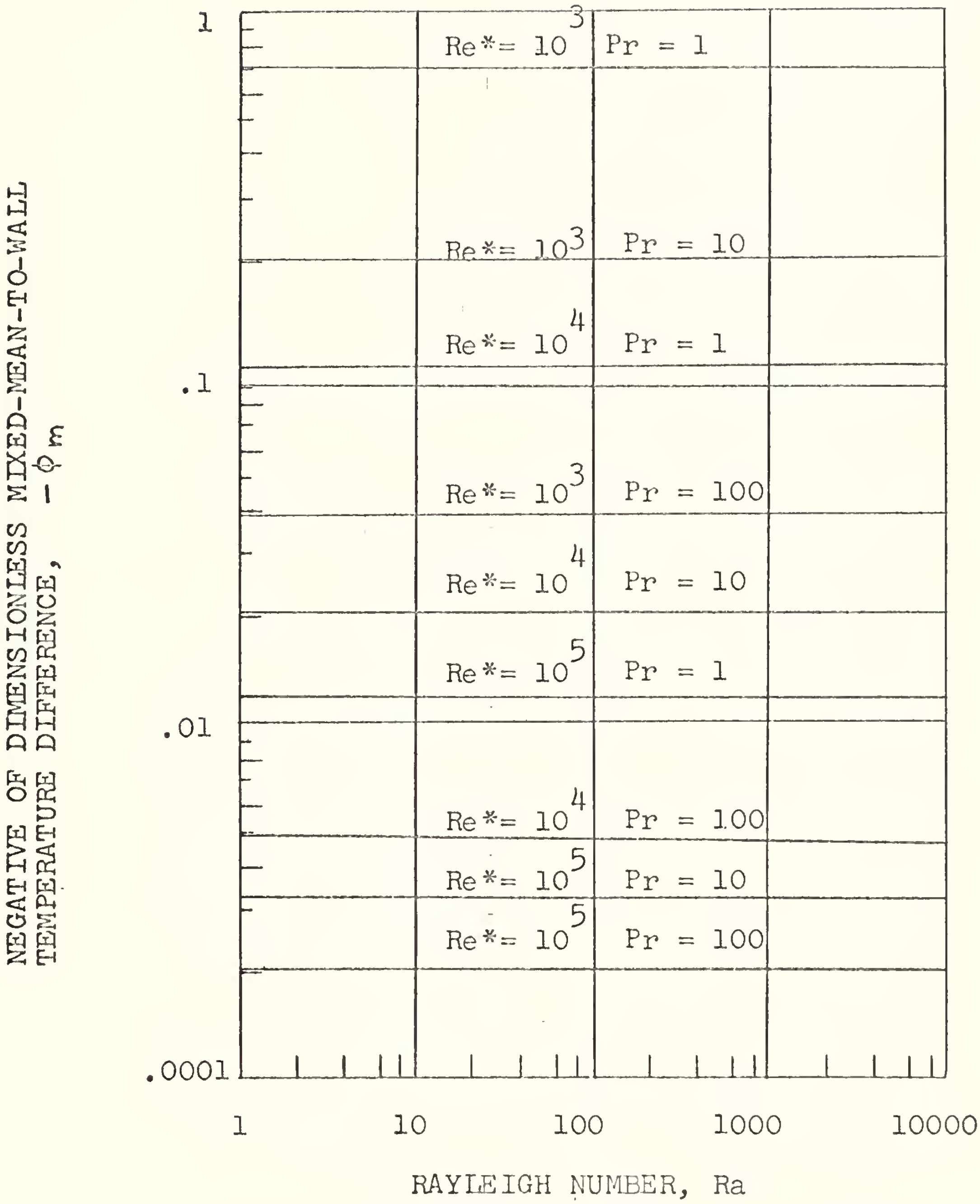

Figure 7. Variation of Negative Mean Temperature Difference with Rayleigh Number for F = 10

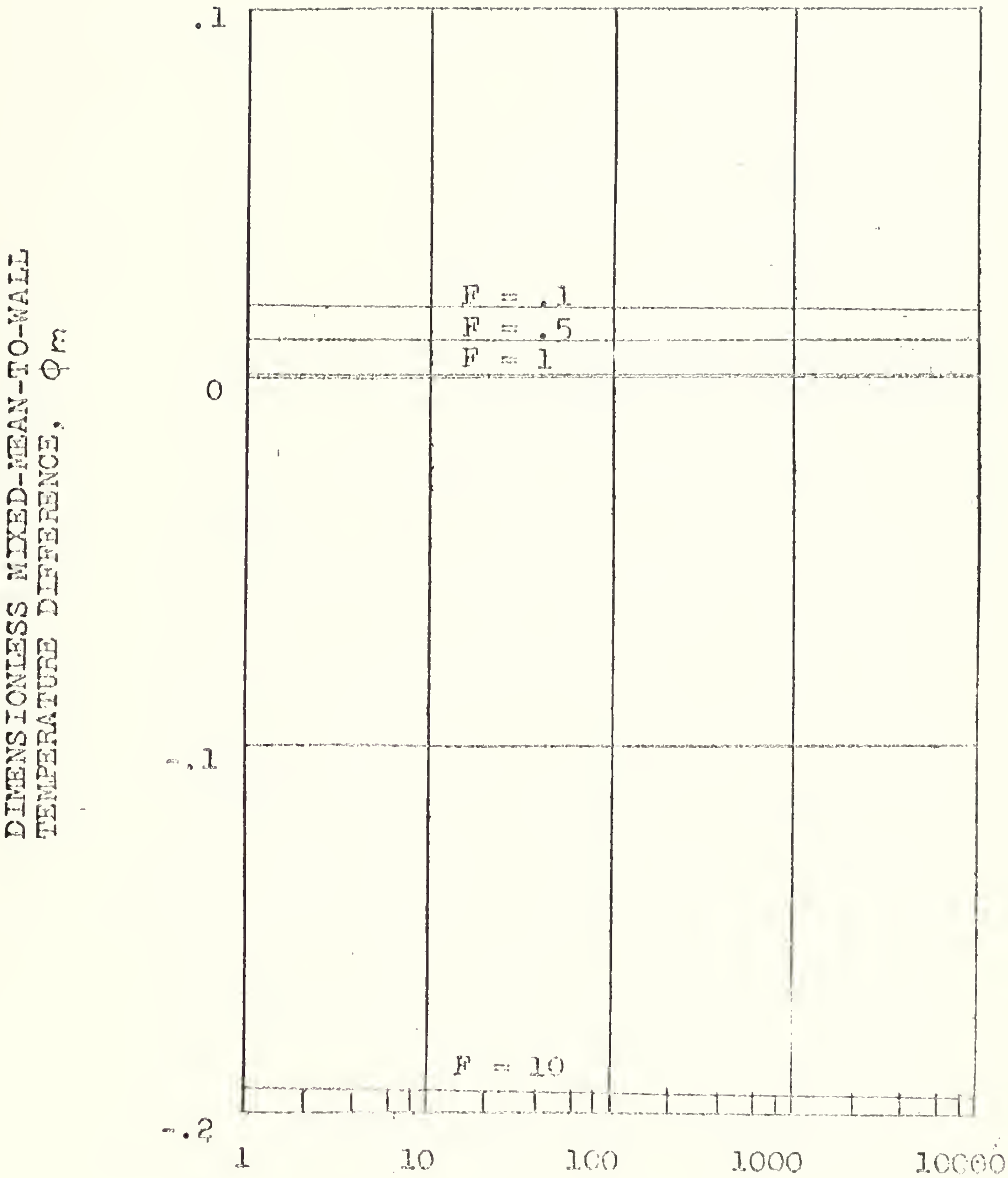

Figure 8. Variation of Mean Temperature Difference with Rayleigh Number for $Re^* = 10^3$, and $Pr = 10$

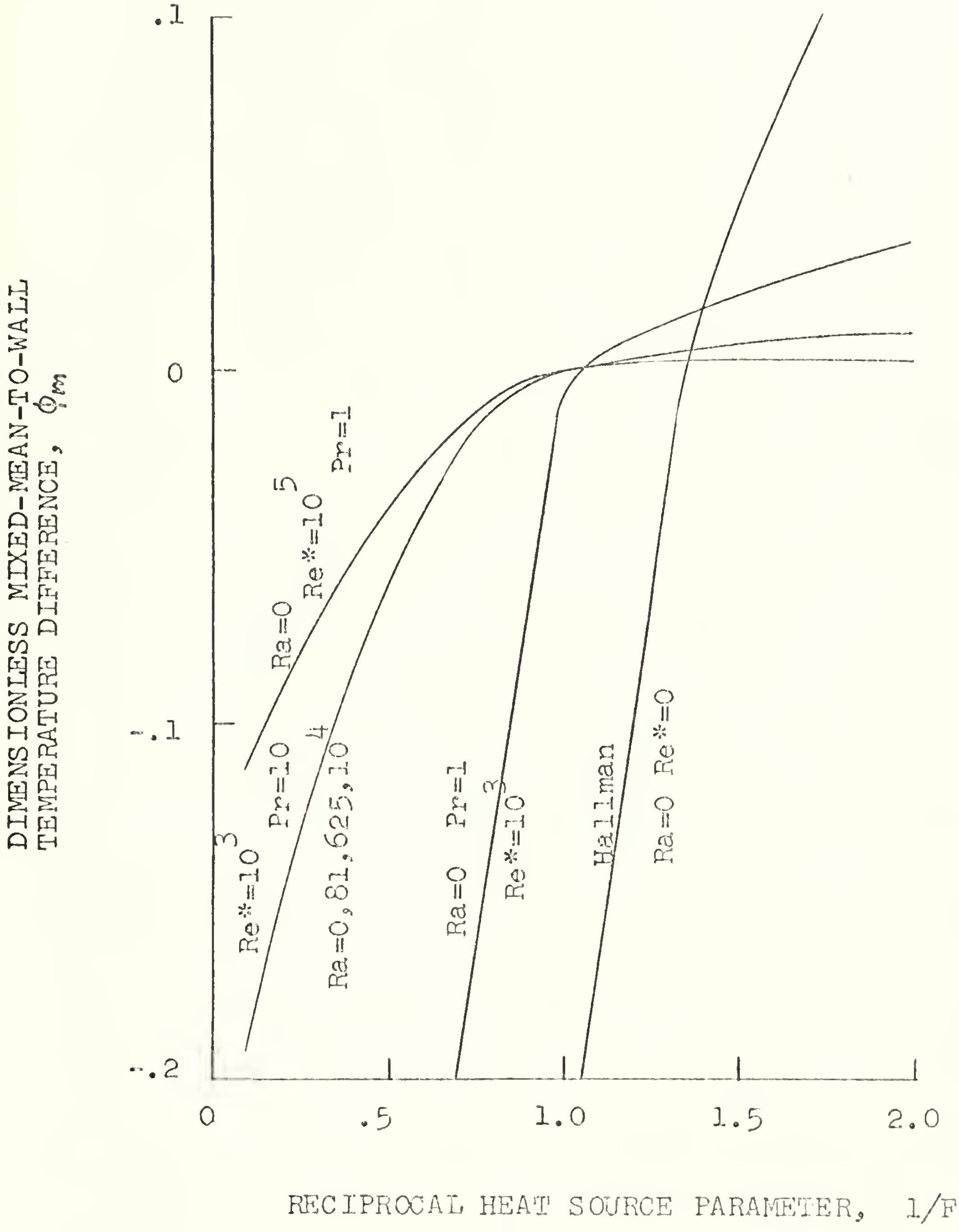

Figure 9. Variation of Mean Temperature Difference with Reciprocal Heat Source Parameter

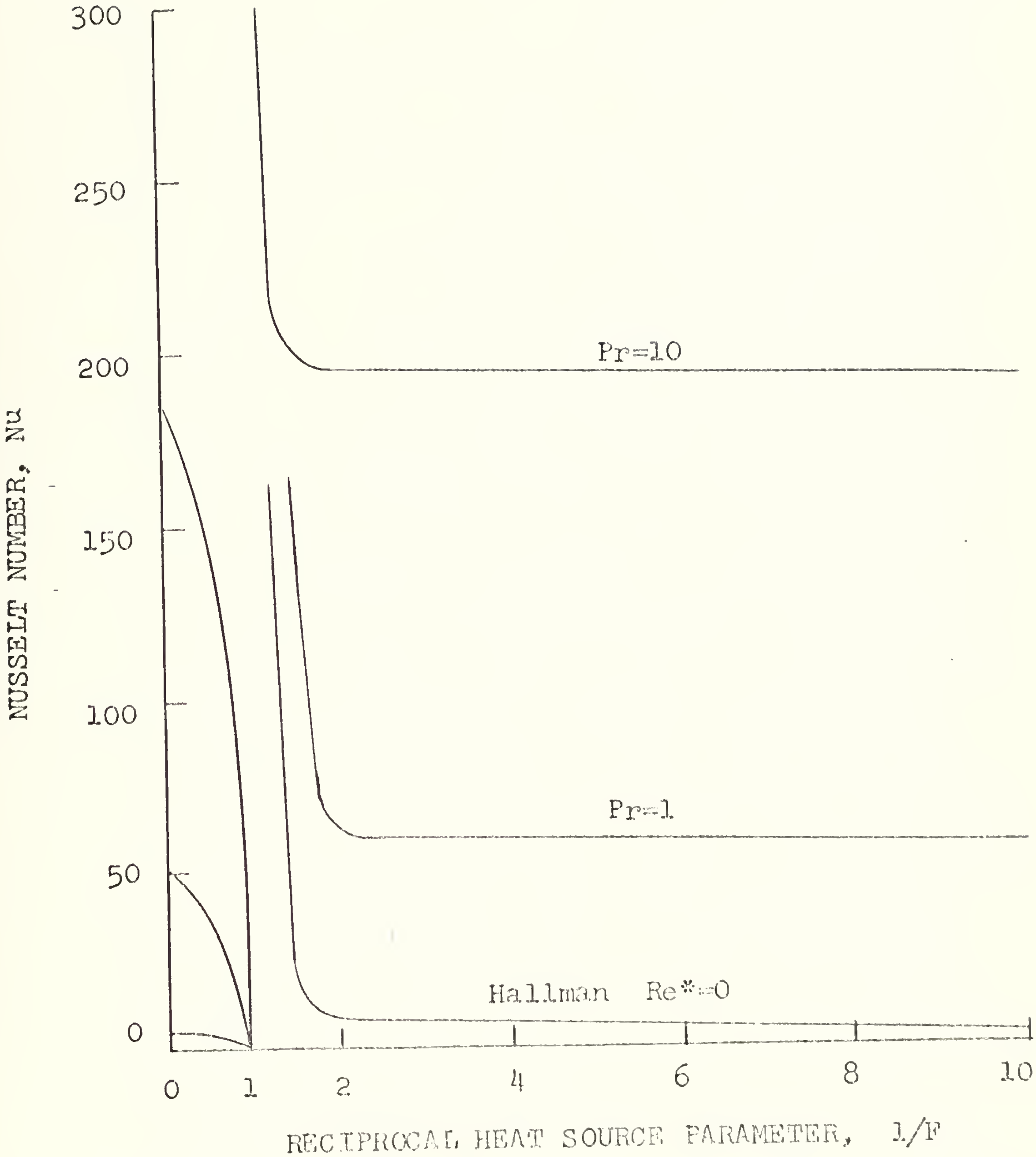

Figure 10. Variation of Nusselt Number with Reciprocal Heat Source Parameter for $\bar{Ra} = 0$ and $Re^* = 10^3$

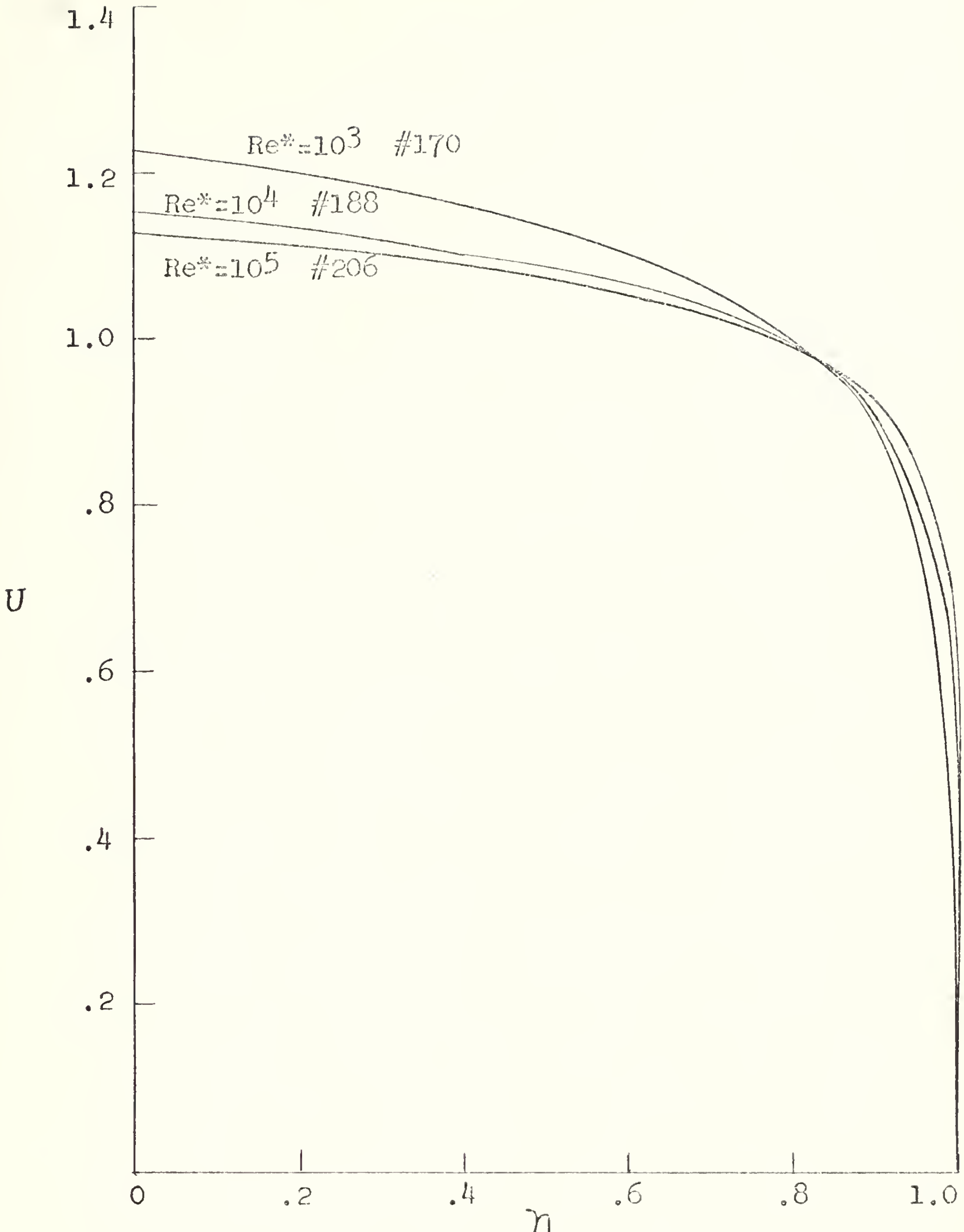

Figure 11. Velocity Profiles, F=5, Pr=10, Ra=625

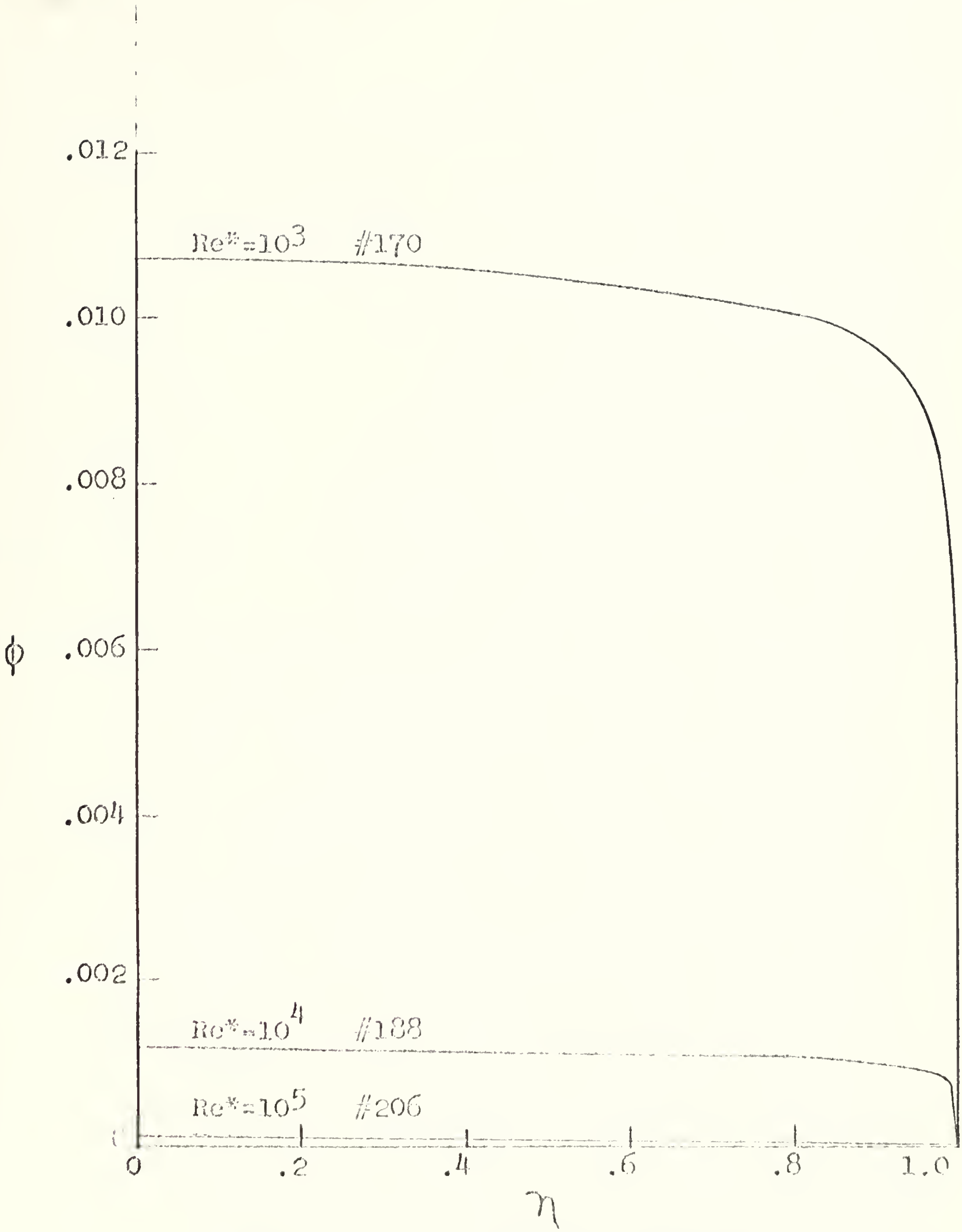

Figure 12. Temperature Difference Profiles, F=5, Pr=10, Ra=625

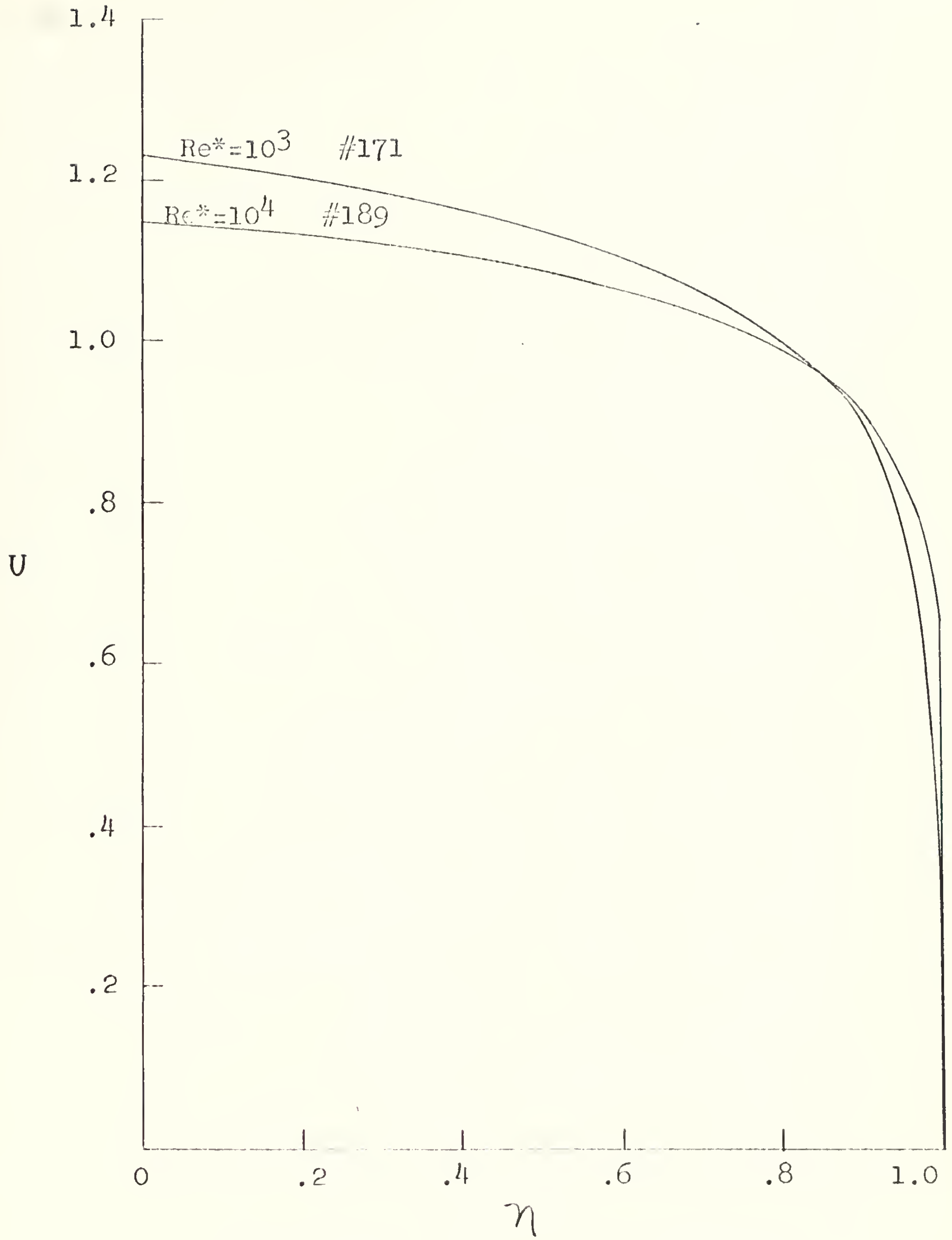

Figure 13. Velocity Profiles, F=1, Pr=10, Ra=625

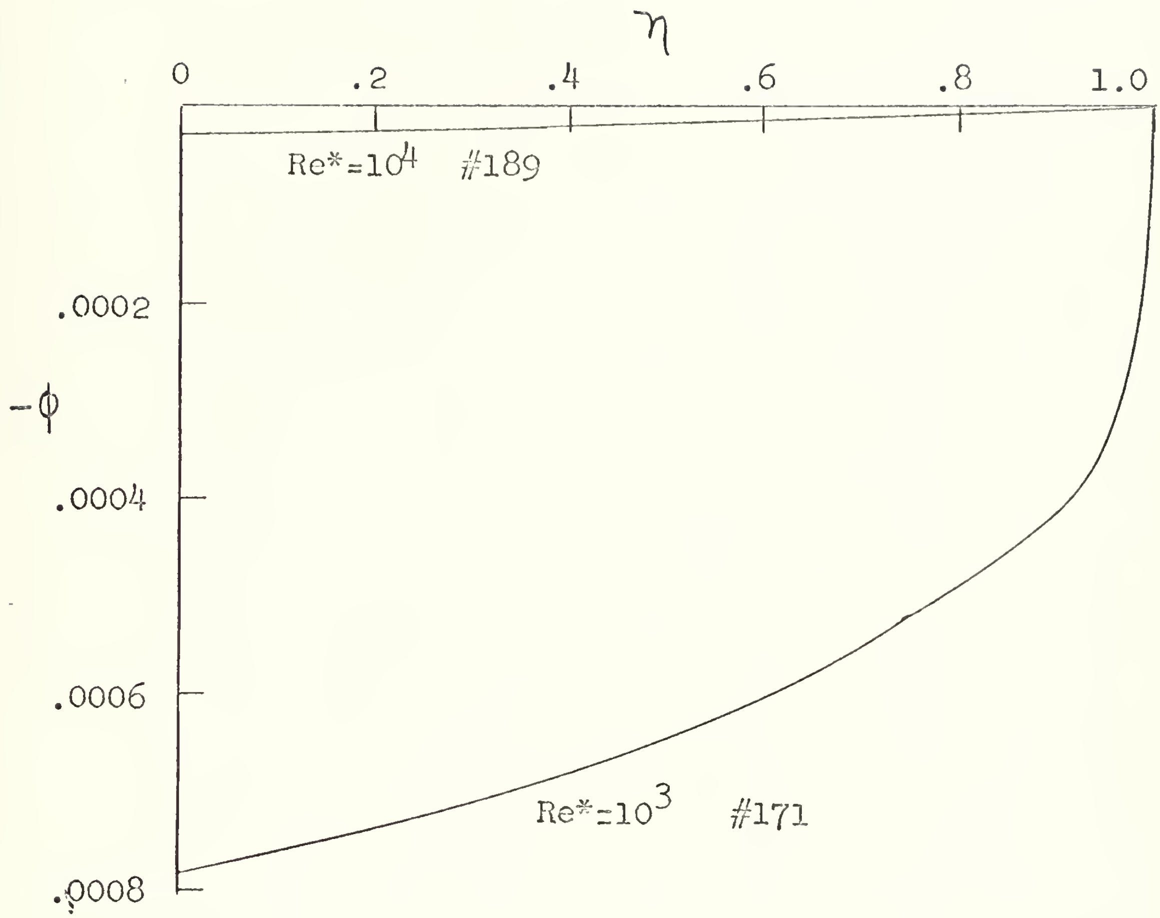

Figure 14. Temperature Difference Profile, F=1, Pr=10, Ra=625

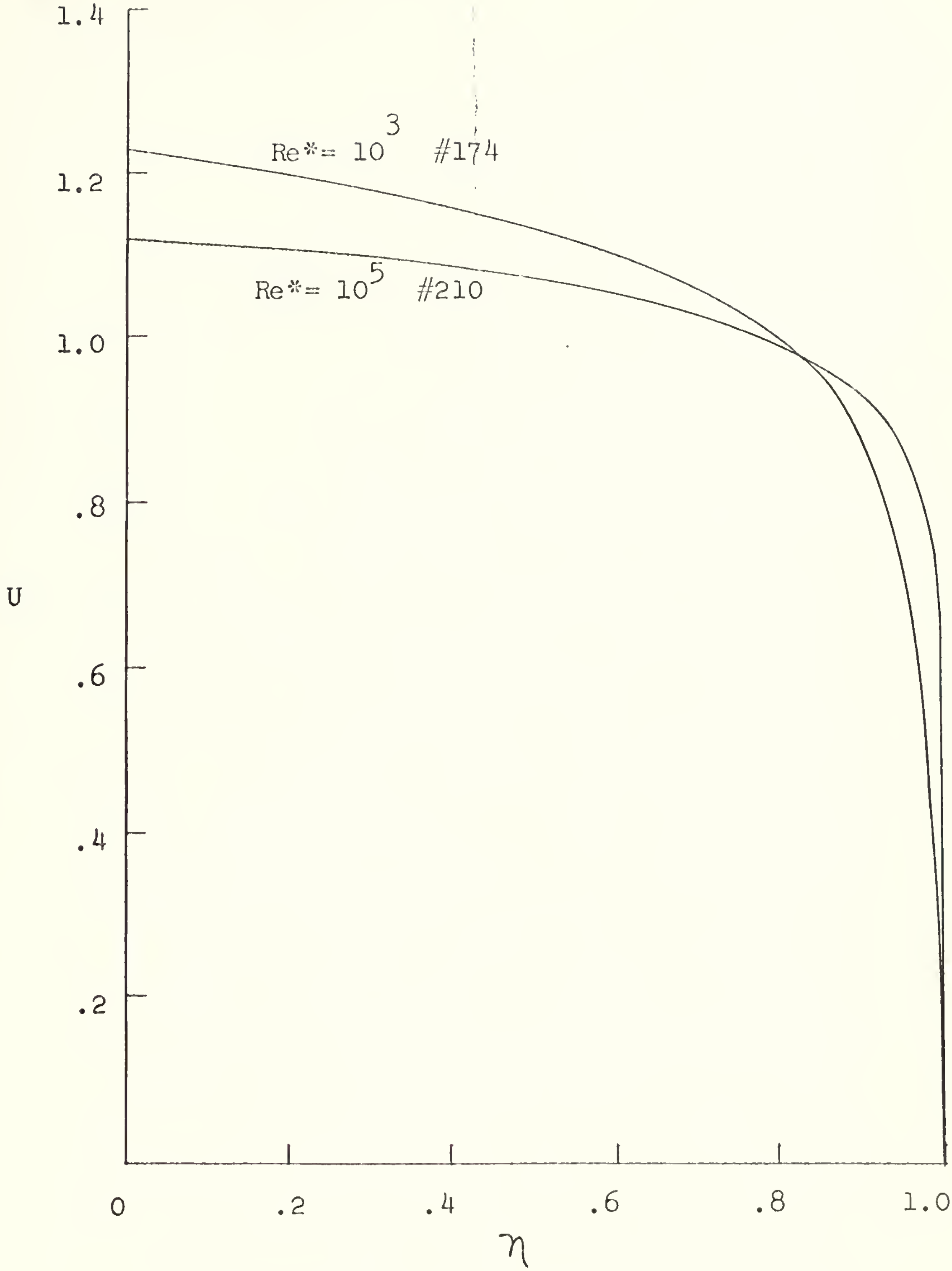

Figure 15. Velocity Profiles, F = 10, Pr = 10, Ra = 625

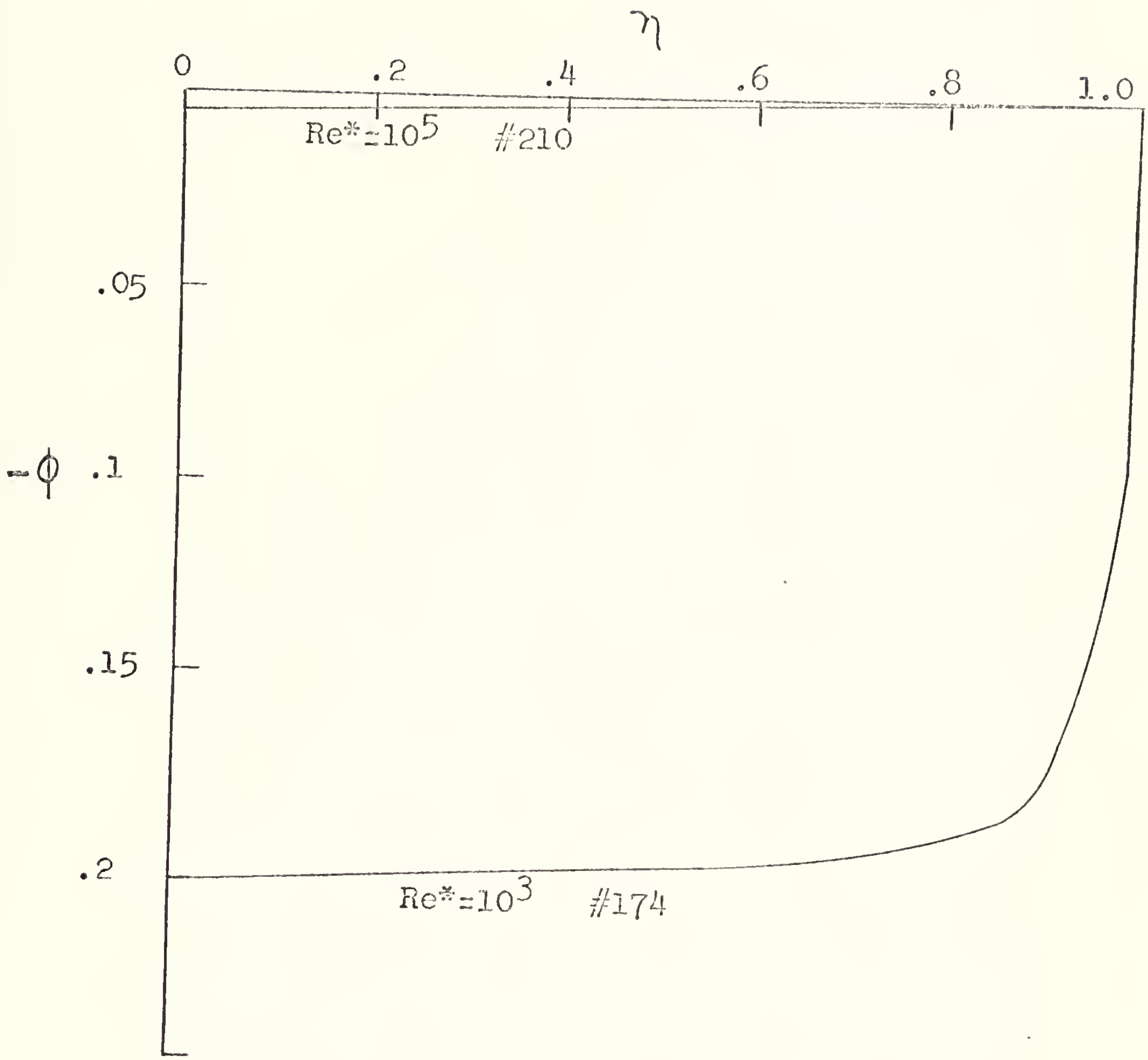

Figure 16. Temperature Difference Profile, F=10, Pr=10, Ra=625

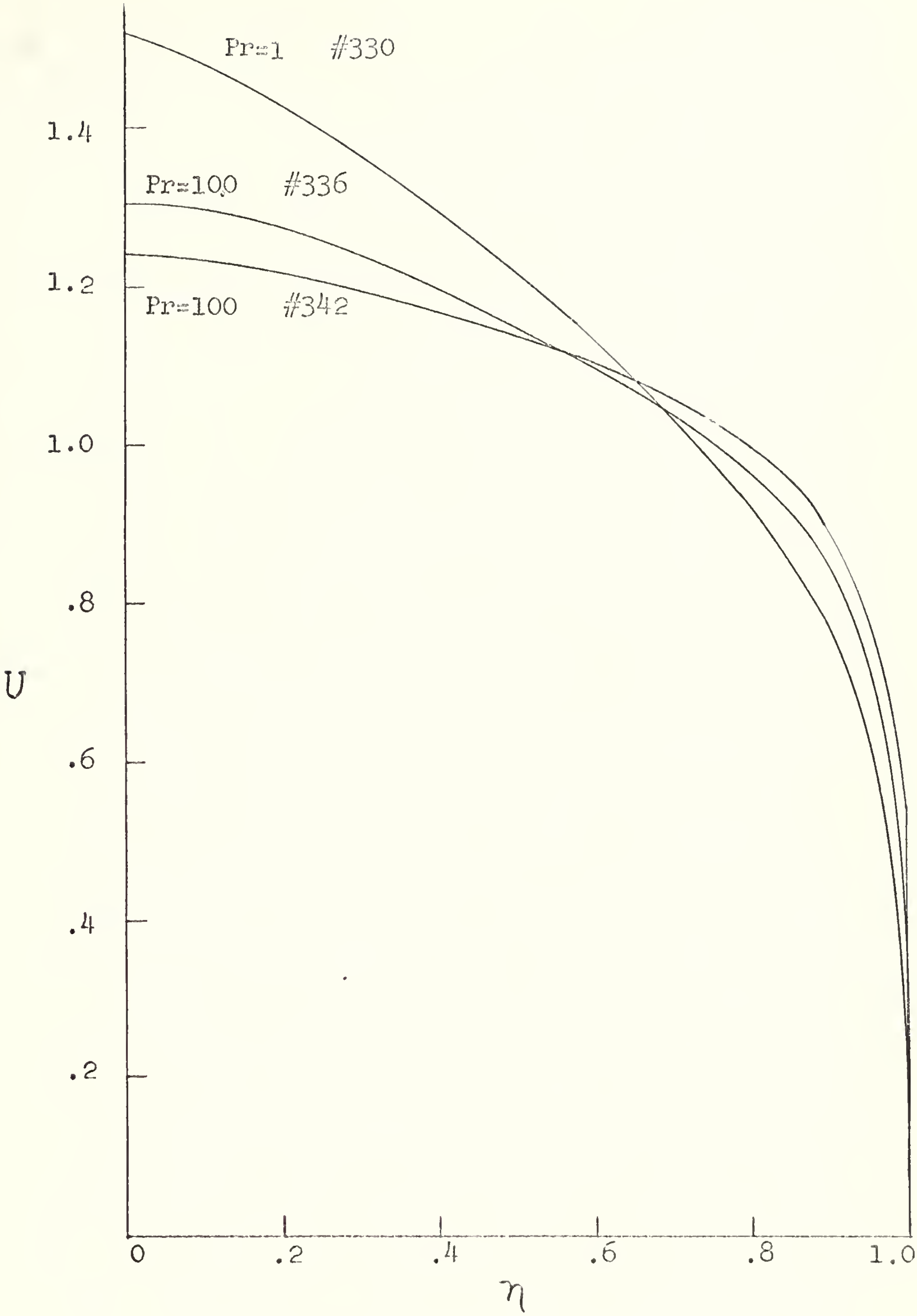

Figure 17. Velocity Profiles, F=10, $Re^*=10^3$, $Ra=10^4$

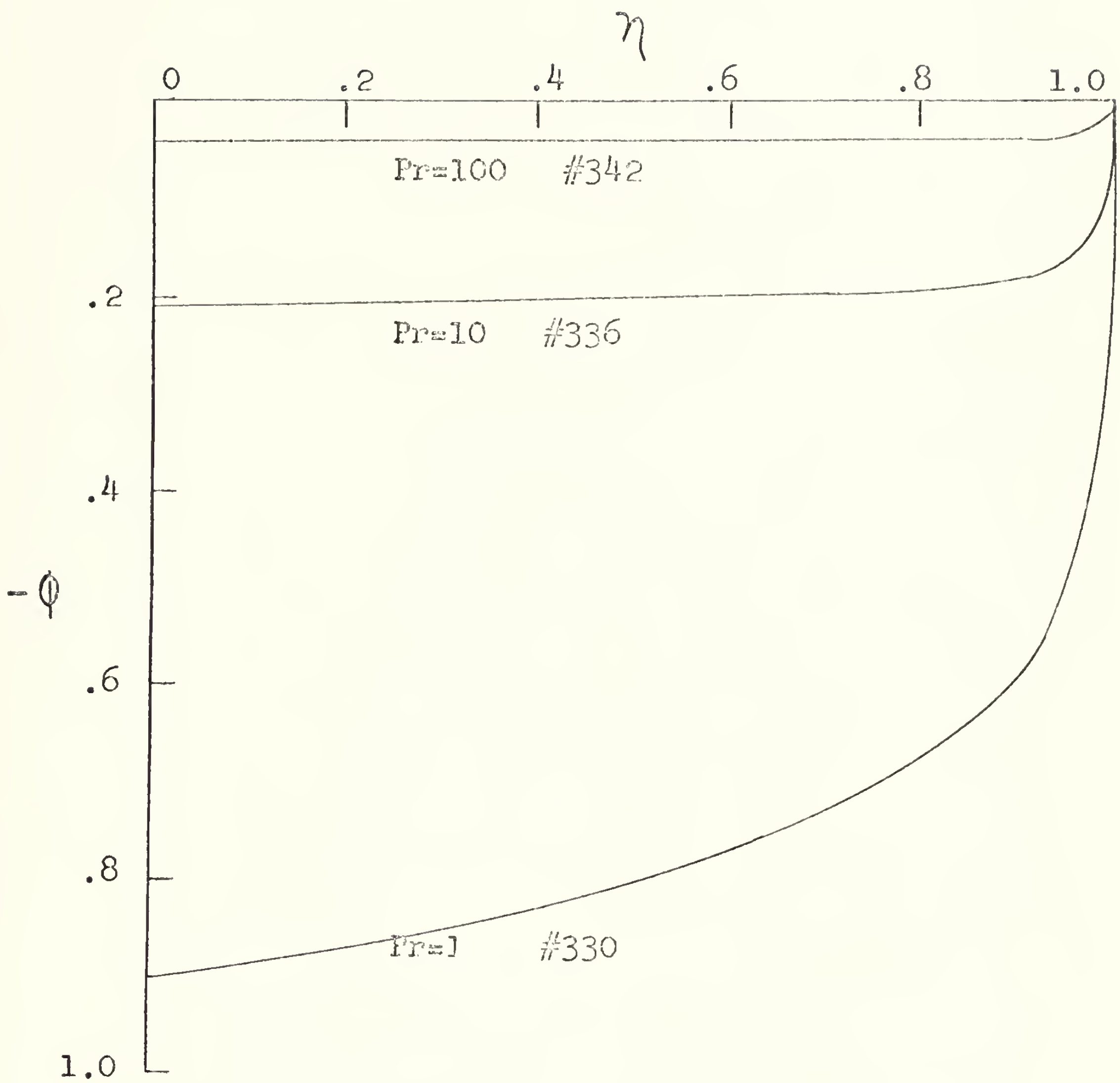

Figure 18. Temperature Difference Profiles, F=10, $Re^*=10^3$, $Ra=10^4$

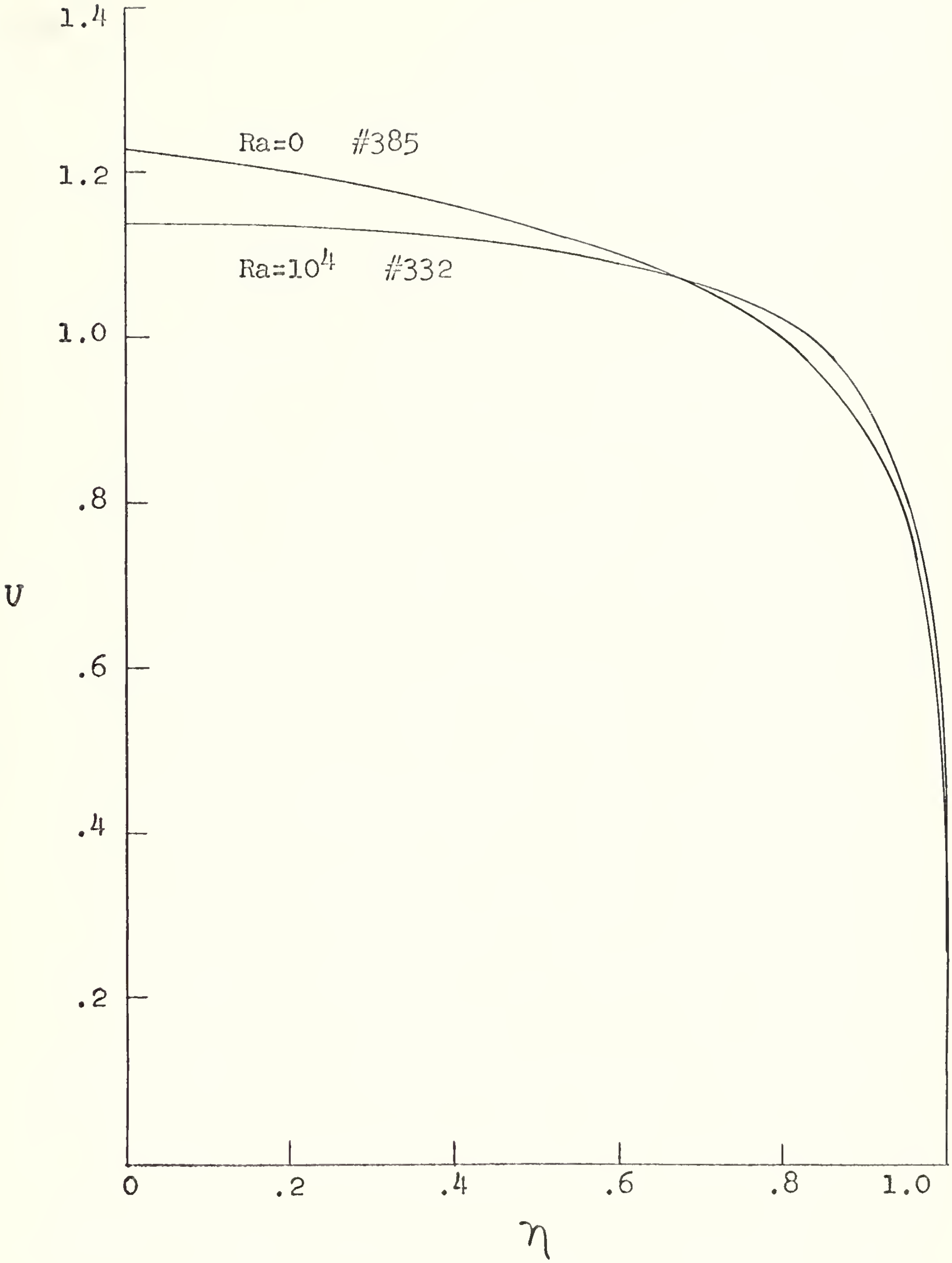

Figure 19. Velocity Profiles, F=5, Pr=10, $Re^{*}=10^{3}$

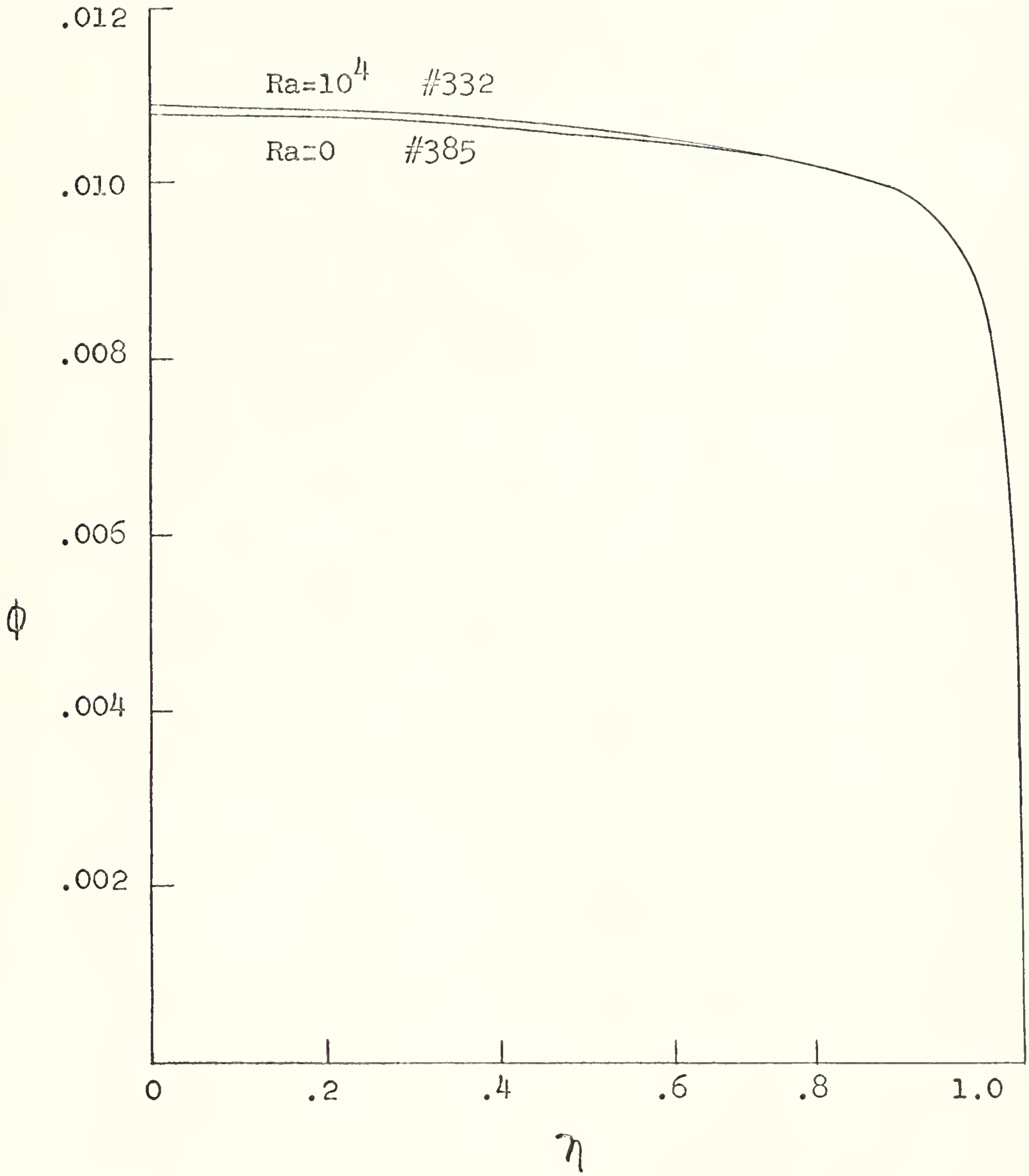

Figure 20. Temperature Difference Profile, $F=.5$, $Pr=10$, $Re^*=10^3$

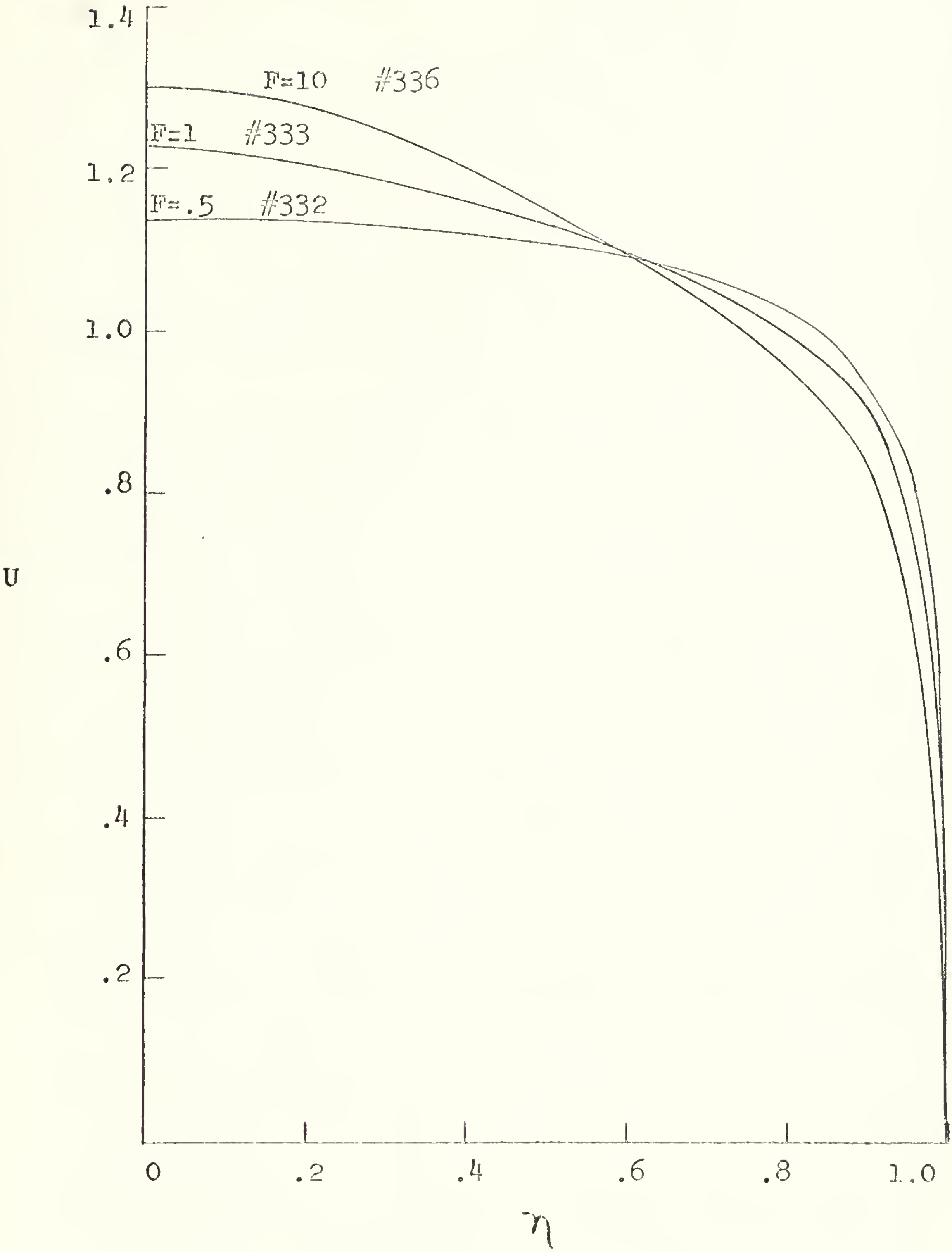

Figure 21. Velocity Profiles, Pr=10, $Re^{*}=10^{3}$, $Ra=10^{4}$

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APPENDIX A

NOMENCLATURE

A   axial temperature gradient in fluid, $\frac{\partial t}{\partial x}$, °F/ft

C   pressure drop parameter, $-\left(\frac{dp}{dx} + \rho_w g\right) D^2 / 32 \mu u_m$, dimensionless

c_p specific heat of fluid at constant pressure, $\frac{BTU}{lb_m °F}$

D   tube inside diameter, ft

g   acceleration due to gravity, ft/sec^2

g_c dimensional constant in Newton's Law,

4.17×10^8 ft $lb_m/lb_f hr^2$

h   convection heat transfer coefficient,
$q''_w / -\theta_m$, BTU/hr ft °F

k   thermal conductivity of fluid, BTU/hr ft °F

p   static fluid pressure, lb_f/ft^2 (absolute)

q_w'' heat transfer rate per unit area at the wall,

BTU/hr ft^2

q''' volume heat sources, BTU/hr ft^3

r   radial coordinate, D/2 - y, ft

t   static fluid temperature, °F

Δt^+ dimensionless temperature difference,

$(t - t_w)\,\rho_m c_p \sqrt{\tau_w / \rho_m} \,/\, q_w''$

T   time, hr

u	fluid velocity parallel to tube axis at radius r, ft/hr
u^+	dimensionless velocity, $u/\sqrt{\tau_w/\rho_w}$
U	dimensionless velocity, u/u_m
u^*	friction velocity, $\sqrt{\tau_w g_c/\rho_w}$, ft/hr
v	specific volume, ft^3/lb_m
$\bar{V}$	velocity vector, ft/hr
x	distance measured along the axis of the tube upwards, ft
y	radial coordinate measured from the wall, D/2 - r, ft
y^+	dimensionless distance from wall, $y\sqrt{\tau_w/\rho_w}/\nu$
α	thermal diffusivity, $\frac{k}{\rho c_p} = \frac{\nu}{Pr}$, ft^2/hr
ϵ_H	eddy diffusivity of heat transfer, ft^2/hr
ϵ_M	eddy diffusivity of momentum transfer, ft^2/hr
η	dimensionless radius, 2r/D
θ	radial temperature difference, $t - t_w$, °F
μ	dynamic viscosity of fluid, lb_m /ft hr
ν	kinematic viscosity of fluid, ft^2/hr
ρ	mass density of fluid, lb_m / ft^3
σ	ratio of eddy diffusivities, $\frac{\epsilon_H}{\epsilon_M}$, dimensionless
τ	fluid shear stress, lb_f / ft^2
ϕ	dimensionless temperature difference, $\frac{16K\theta}{\rho_m u_m c_p A D^2}$
ψ	angular coordinate of the cylindrical coordinate system, radians

SUBSCRIPTS

H heat

m mean

M momentum

r based on radial position

w radial position of wall of tube

x based on axial position

ψ based on angular position

SUPERSCRIPTS

$'$ differentiation once with respect to the independent variable, η

$''$ per unit area

$'''$ per unit volume

DIMENSIONLESS NUMBERS

Gr Grashof number, $\frac{\rho_m \rho_w \beta g A D^4}{16 \mu^2}$

Nu Nusselt number, hD/k

Pr Prandtl number, $c_p \mu / k = \nu / \alpha$

Ra Rayleigh number, $\frac{\rho_m \rho_w c_p A D^4}{16 \mu k} = Gr\, Pr$

Re Reynolds number, $\rho u_m D / \mu = u_m D / \nu$

Re* Friction Reynolds number, Du^* / ν

APPENDIX B

THE EDDY DIFFUSIVITY OF MOMENTUM

An analytical study was made by Jackson[4] of the eddy diffusivity of momentum in a vertical circular tube. The ratio of eddy diffusivity to kinematic viscosity, $\frac{\epsilon_M}{\nu}$, called relative viscosity appeared in a reduced form of the Navier-Stokes equation. An empirical fit of experimental data was employed to determine the dimensionless velocity gradient appearing in the expression for $\frac{\epsilon_M}{\nu}$. 1,11

The result of his study used in the present analysis is the following set of expressions for variation of relative viscosity with dimensionless radius .

$$\frac{\epsilon_M}{\nu} = \frac{\eta}{1-.005\,Re^*(1-\eta)[41/9-.025\,Re^*(1-\eta)]} - 1 \qquad \text{for } 1-\frac{60}{Re^*} \leq \eta \leq 1$$

$$\frac{\epsilon_M}{\nu} = .2\,Re^*\eta(1-\eta) - 1 \qquad \text{for } \frac{1}{10} \leq \eta < 1-\frac{60}{Re^*}$$

$$\frac{\epsilon_M}{\nu} = 9\,Re^*/500 - 1 \qquad \text{for } 0 \leq \eta < \frac{1}{10}$$

The results compare favorably with the experimental and analytical results of others and appears to be an improvement over the $\frac{\epsilon_M}{\nu}$ expression used in recent studies. These results appear as equations (29), (30), and (31) in the present analysis.

APPENDIX C

THE LONGITUDINAL TEMPERATURE GRADIENT

The heat balance for the steady-state condition in our problem states that the rate at which heat is added to the fluid is equal to the rate at which heat is convected downstream in the fluid. In symbolic form (see Figure C-1),

$$q''' \frac{\pi D^2}{4} dx - q_w'' \pi D dx = \left[\rho_m \int_0^{D/2} u 2\pi r dr\right] c_p \frac{\partial t}{\partial x} dx \qquad \text{(C-1)}$$

where q_w'' is taken positive if heat is being removed. Solving equation (C-1) for $\partial t / \partial x$ gives

$$\frac{\partial t}{\partial x} = \frac{q''' D^2/4 - q_w'' D}{2 c_p \rho_m \int_0^{D/2} u r dr} \quad . \qquad \text{(C-2)}$$

The mean velocity is defined as follows:

$$u_m \equiv \frac{\int_0^{D/2} u 2\pi r dr}{\pi D^2/4} = \frac{8 \int_0^{D/2} u r dr}{D^2} \qquad \text{(C-3)}$$

from which

$$\int_0^{D/2} u r dr = \frac{u_m D^2}{8} \quad . \qquad \text{(C-4)}$$

Equation (C-2) can therefore be written as

$$\frac{\partial t}{\partial x} = \frac{q''' D - 4 q_w''}{\rho_m c_p u_m D} \quad . \qquad \text{(C-5)}$$

Since all quantities on the right-hand side of equation (C-5) are constant in our problem,

$$\frac{\partial t}{\partial x} = A \qquad \text{(a constant)} \qquad \text{(C-6)}$$

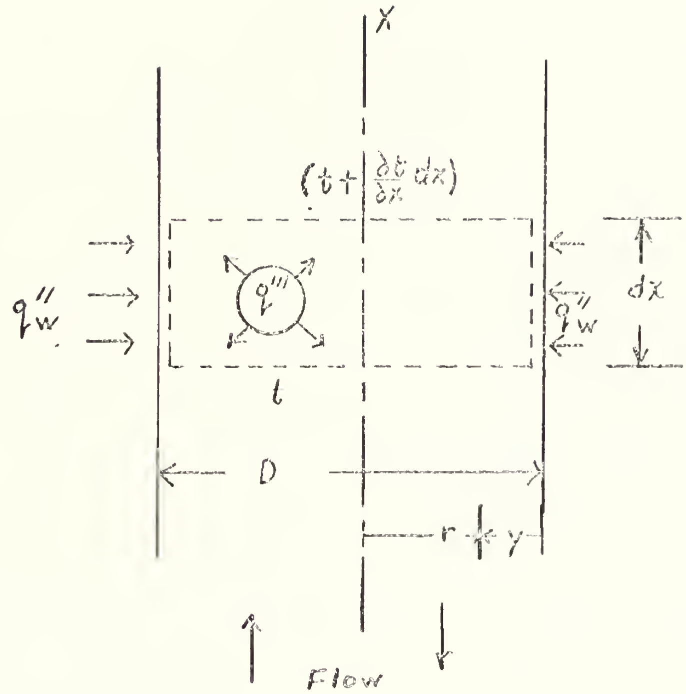

FIGURE C-1

Heat Balance for an Element of Fluid

Therefore, the temperature gradient in the direction of flow is a constant independent of radial location.

APPENDIX D

NUSSELT NUMBER

Nusselt number Nu is a useful expression in comparing our problem results. It is function of the temperature gradient at the wall to a constant average temperature gradient[5].

Considering a heat balance on a unit length of tube:

$$q''' \frac{\pi D}{4} - q_w'' \pi D = \rho_m u_m c_p A \frac{\pi D^2}{4} \qquad \text{(D-1)}$$

where q_w'' is taken positive if heat is being removed[3]. Substituting in equation (D-1) the equations

$$q_w'' = h\,\theta_m$$

and

$$\phi_m = \frac{16 k \theta_m}{q''' D^2}$$

gives

$$\frac{q'''}{\rho_m u_m c_p A} - \frac{hD}{k} \frac{\phi_m q'''}{4 \rho_m u_m c_p A} = 1 \qquad \text{(D-2)}$$

Substituting the definition of F in equation (D-2) and rearranging terms gives

$$Nu \equiv hD/k = \frac{4}{\phi_m}(1-F) \qquad \text{(D-3)}$$

From this equation, as ϕ_m approaches zero, Nu will approach infinity; and when F = 1, Nusselt number will equal zero.

APPENDIX E

REYNOLDS NUMBER

Reynold number Re is the ratio of the inertia forces to frictional forces and is a measure of turbulence.

Considering a dynamic force balance on an element of fluid, the following expression can be developed:

$$\frac{\partial p}{\partial x} = \frac{dp}{dx} = \mp 4 \frac{\tau_w}{D} - \rho_m \frac{g}{g_c} \tag{E-1}$$

where + is for upward flow and - is for downward flow[8].

From equation (13) the pressure-drop parameter C is

$$C = \frac{-D^2 g_c \left(\frac{dp}{dx} + \rho_w \frac{g}{g_c} \right)}{32 \mu u_m} \tag{E-2}$$

and substituting equation (E-1) in equation (E-2) gives

$$C = \frac{-D^2 g_c \left[\mp \frac{4\tau_w}{D} + (\rho_w - \rho_m) \frac{g}{g_c} \right]}{32 \mu u_m} \tag{E-3}$$

Using equations

$$\tau_w = (u^*)^2 \rho_w / g_c \quad ;$$

$$\rho_w - \rho_m = \rho_w \beta \theta_m \quad ;$$

$$\theta_m \equiv q''' D^2 \phi_m / 16K \quad ;$$

and the definitions of Re, Re*, Ra, and F, we obtain

$$C = \frac{\pm (Re^*)^2}{8 Re} - \frac{Ra\, \phi_m}{32 F} \, . \tag{E-4}$$

Solving for Re gives

$$Re = \frac{\pm (Re^*)^2}{8C + Ra\, \phi_m / 4F} \tag{E-5}$$

which is equation (36) in the Analysis.

APPENDIX F

VOLUME HEAT SOURCES

The term q''' accounts for the volume heat sources in our problems. Since only uniform volume heat sources are treated, q''' is independent of direction.

The nondimensional volume heat source parameter is

$$F \equiv \frac{\rho_m c_p u_m A}{q'''} \tag{F-1}$$

as defined by Hallman[3]. The physical significance of F can be described as the ratio of the thermal energy convected downstream, per unit volume to the heat generated in the fluid per unit volume. Considering a heat balance on a unit length of pipe with constant wall heat addition

$$q''' \frac{\pi D^2}{4} - q''_w \pi D = \rho_m u_m c_p A \pi \frac{D^2}{4} \tag{F-2}$$

and introducing equation (F-1) gives

$$F = 1 - \frac{4 q''_w}{D q'''} \quad . \tag{F-3}$$

The meaning of the various values of F can be seen from equation (F-3). When $q''_w = 0$, which corresponds to an insulated tube wall, then F = 1. The walls should be hotter than the bulk of the fluid because heat is convected away from this

region less rapidly than in the center, and because of uniform volume heat source generation. Fluid velocities should therefore tend to be higher in the vicinity of the wall than near the center line.

When $F < 1$, it means that heat is being removed at the walls because more heat is being generated internally than is convected downstream. The velocity at the wall should be slower than the velocity at the center line in this case since heat is being removed and the density of the fluid is higher at the wall.

When $F > 1$, it means that heat is being added at the walls because heat is being convected downstream faster than it is being generated. The velocity in this case should rise faster near the walls than near the center line since it is hotter at the walls.

$F = 0$ will occur whenever there is no net through-flow ($u_m = 0$), no flow ($u = 0$), or constant wall temperature ($A = 0$). An investigation is being conducted on the general problem with a constant wall temperature by Page[17].

$F = \infty$ would correspond to no volume heat sources. An investigation is being conducted on this general problem by Griffin[16].